Quiz 4 MTH335 Fall 2026

Sunday, September 13, 2026 9:16 AM

Solve

$$y' = (t^3+1)(3y+7)$$

<u>Soln</u>: This is separable:

$$\frac{dy}{dt} = (t^3+1)(3y+7)$$

separate variables by "multiplying" by dt and dividing by $3y+7$

$$\frac{1}{3y+7}\,dy = (t^3+1)\,dt$$

"integrate"

$$\underbrace{\int \frac{1}{3y+7}\,dy}_{} = \underbrace{\int t^3+1\,dt}_{}$$

$u = 3y+7$

$du = 3\,dy$

$\frac{1}{3}du = dy$

$$= \frac{1}{3}\int \frac{1}{u}\,du \qquad = \frac{t^4}{4} + t + C$$

$$= \frac{1}{3}\ln(|u|)$$

$$= \frac{1}{3}\ln(|3y+7|)$$

Therefore, we have shown

$$\frac{1}{3}\ln(|3y+7|) = \frac{t^4}{4} + t + C$$

$$\ln(|3y+7|) = \frac{3t^4}{4} + 3t + \tilde{C}\ ; \quad \tilde{C} := 3C$$

$$|3y+7| = \hat{C}\,e^{\frac{3}{4}t^4+3t}\ ; \quad \hat{C} = e^{\tilde{C}}$$

I used property $e^{a+b} = e^a e^b$ here!

$$3y+7 = \bar{C}\,e^{\frac{3}{4}t^4+3t}\ ; \quad \bar{C} = \pm\hat{C}$$

"remove" absolute value by using $\pm$

for ex,

$|x| = 5$ means $x = 5$ or $x = -5$

solve for y

$$\boxed{y = \check{C}\,e^{\frac{3}{4}t^4+3t} - 7}\ ; \quad \check{C} = \frac{\bar{C}}{3}$$