

Quiz 7 MTH 428/528 Spring 2025

Monday, April 21, 2025

11:42 AM

Prove $\int_{\underline{}} f+g \, dV \geq \int_{\underline{}} f \, dV + \int_{\underline{}} g \, dV$

Proof: Given any $\epsilon > 0 \exists$ step function ϕ_1 s.t. $\phi_1 \leq f$ and

$$\int \phi_1 \, dV > \int_{\underline{}} f \, dV + \frac{\epsilon}{2}$$

Since $\int_{\underline{}} f$ is the sup of all such ϕ , !!

and $\exists$ step fct ϕ_2 s.t. $\phi_2 \leq g$ and

$$\int \phi_2 \, dV > \int_{\underline{}} g \, dV + \frac{\epsilon}{2}$$

Now, $\phi_1 + \phi_2$ is step fct w/ $\phi_1 + \phi_2 \leq f+g$ and

$$\int \phi_1 + \phi_2 \, dV \leq \int_{\underline{}} f+g \, dV$$

Since $\int \phi_1 + \phi_2 \, dV = \int \phi_1 \, dV + \int \phi_2 \, dV$, we get

$$\int_{\underline{}} f+g \, dV \geq \int \phi_1 + \phi_2 \, dV = \int \phi_1 \, dV + \int \phi_2 \, dV > \int_{\underline{}} f \, dV + \int_{\underline{}} g \, dV + \epsilon$$

Since this holds $\forall \epsilon > 0$, we obtain

$$\int_{\underline{}} f+g \, dV \geq \int_{\underline{}} f \, dV + \int_{\underline{}} g \, dV$$