Quiz 6 MTH 428/528 Spring 2025

Monday, April 21, 2025 11:31 AM

Prove

$$\underline{V}(A \cup B) \geq \underline{V}(A) + \underline{V}(B)$$

Proof: Let $\epsilon > 0$. $\exists$ cpt sets $K \subset A$ and $L \subset B$ so that $K \cap L = \emptyset$ and

$(\star)$ $\quad V(K) > \underline{V}(A) + \frac{\epsilon}{2}$ and $V(L) > \underline{V}(B) + \frac{\epsilon}{2}$

Then $K \cup L$ is compact and $K \cup L \subset A \cup B$, so by Lem 4,

$$\underline{V}(A \cup B) \underset{\substack{\text{inner measure} \\ \text{is sup of} \\ \text{measures of} \\ \text{cpt subsets}}}{\geq} \underline{V}(K \cup L) \underset{\text{lem 4}}{\geq} \underline{V}(K) + \underline{V}(L) \underset{(\star)}{>} \underline{V}(A) + \underline{V}(B) + \epsilon$$

We have since ϵ arbitrary,

$$\underline{V}(A \cup B) \geq \underline{V}(A) + \underline{V}(B)$$