

Quiz 1 MTH 428.528 Spring 2025

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hyperplane in E^4 containing the vectors

$$e_1 - e_2, 2e_2, e_3 + e_4, \text{ and } e_1 - 3e_4$$

Need to use the definition of a hyperplane: $\{x \in E^4 : x \cdot \mathbf{z} = c\}$

unknowns

So check the condition for some unknown constant c

$$\text{and unknown vector } \mathbf{z} = (z^1, z^2, z^3, z^4)$$

for our four vectors:

$$\begin{aligned}(e_1 - e_2) \cdot \mathbf{z} &= [(1, 0, 0, 0) - (0, 1, 0, 0)] \cdot (z^1, z^2, z^3, z^4) \\ &= (1, -1, 0, 0) \cdot (z^1, z^2, z^3, z^4) \\ &= z^1 - z^2\end{aligned}$$

$$2e_2 \cdot \mathbf{z} = 2z^2$$

$$(e_3 + e_4) \cdot \mathbf{z} = z^3 + z^4$$

$$(e_1 - 3e_4) \cdot \mathbf{z} = z^1 - 3z^4$$

Setting each dot product equal to c gives us the system

$$\begin{cases} z^1 - z^2 = c \\ 2z^2 = c \\ z^3 + z^4 = c \\ z^1 - 3z^4 = c \end{cases}$$

Diagram showing the solution process:

- From $z^1 - z^2 = c$, we get $z^2 = z^1 - c$.
- From $2z^2 = c$, we get $2(z^1 - c) = c$, which simplifies to $2z^1 - 2c = c$, leading to $z^1 = \frac{3}{2}c$.
- From $z^1 - 3z^4 = c$, we get $z^4 = \frac{z^1 - c}{3} = \frac{\frac{3}{2}c - c}{3} = \frac{c}{6}$.
- From $z^3 + z^4 = c$, we get $z^3 + \frac{c}{6} = c$, leading to $z^3 = \frac{5c}{6}$.
- From $2z^2 = c$, we get $z^2 = \frac{c}{2}$.

c is free variable... pick $c=6$ to get rid of fractions

$$\Rightarrow \mathbf{z} = (z^1, z^2, z^3, z^4) = \left(\frac{3c}{2}, \frac{c}{2}, \frac{5c}{6}, \frac{c}{6} \right)$$

$$c=6$$

$$\mathbf{z} = (9, 3, 5, 1)$$

So we have shown that the hyperplane

$$\{x \in E^4 : \mathbf{z} \cdot x = 6\}$$

contains the given 4 points