Quiz 1 MTH 428.528 Spring 2025

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hyperplane in E^4 containing the vectors

$e_1 - e_2$, $2e_2$, $e_3 + e_4$, and $e_1 - 3e_4$

Need to use the definition of a hyperplane: $\{x \in E^4 : x \cdot \underbrace{z = c}_{\text{unknowns}}\}$

So check the condition for some unknown constant c

and unknown vector $z = (z^1, z^2, z^3, z^4)$

for our four vectors:

$$(e_1 - e_2) \cdot z = [(1,0,0,0) - (0,1,0,0)] \cdot (z^1, z^2, z^3, z^4)$$
$$= (1,-1,0,0) \cdot (z^1, z^2, z^3, z^4)$$
$$= z^1 - z^2$$

$$2e_2 \cdot z = 2z^2$$

$$(e_3 + e_4) \cdot z = z^3 + z^4$$

$$(e_1 - 3e_4) \cdot z = z^1 - 3z^4$$

Setting each dot product equal to c gives us the system

$$\begin{cases} z^1 - z^2 = c \\ 2z^2 = c \\ z^3 + z^4 = c \\ z^1 - 3z^4 = c \end{cases}$$

$$z^1 - z^2 = c \longrightarrow \boxed{z^2 = z^1 - c}$$

$$2(z^1 - c) = c$$
$$2z^1 - 2c = c$$
$$\boxed{z^1 = \tfrac{3}{2}c}$$

$$\boxed{z^2 = \tfrac{3}{2}c - c = \frac{c}{2}}$$

$$\boxed{z^4 = \frac{z^1 - c}{3} = \frac{\frac{3}{2}c - c}{3} = \frac{c}{6}}$$

$$z^3 + \frac{c}{6} = c$$
$$\boxed{z^3 = \frac{5c}{6}}$$

c is free variable... pick $c = 6$ to get rid of fractions

$$\Rightarrow z = (z^1, z^2, z^3, z^4) = \Big(\underbrace{\frac{3c}{2}, \frac{c}{2}, \frac{5c}{6}}, \frac{c}{6}\Big)$$

$$\Big\downarrow c = 6$$

$$z = (9, 3, 5, 1)$$

So we have shown that the hyperplane

$$\{x \in E^4 : z \cdot x = 6\}$$

Contains the given 4 points