Quiz 9 MTH 335 Fall 2025

Monday, October 13, 2025 12:00 PM

Compute $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s}\right\}$

Soln: Using partial fractions method,

$$\frac{s+1}{s^2+2s} = \frac{s+1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

Multiply by the common denominator:

$$s+1 = A(s+2) + Bs$$

$s=-2$: $-1 = B(-2)$, $B = \frac{1}{2}$

$s=0$: $1 = 2A$, $A = \frac{1}{2}$

Thus we see that

$$\frac{s+1}{s^2+2s} = \frac{1/2}{s} + \frac{1/2}{s+2}$$

Recall from Laplace table that $\mathcal{L}\{1\}(s) = \frac{1}{s}$ and $\mathcal{L}\{e^{at}\}(s) = \frac{1}{s-a}$

Thus we conclude

$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s-(-2)}\right\}$$

$$= \frac{1}{2} + \frac{1}{2}e^{-2t}$$