Quiz 8 MTH 335 Fall 2025

Monday, October 13, 2025 11:51 AM

$$\mathcal{L}\{t\}(z) \overset{\text{def}}{=} \int_0^\infty t e^{-zt}\,dt$$

$u = t \quad dv = e^{-zt}dt$

$du = dt \quad v = -\frac{1}{z}e^{-zt}$

$$= \lim_{b\to\infty} \int_0^b t e^{-zt}\,dt$$

$$\overset{\text{(parts)}}{=} \lim_{b\to\infty} -\frac{t}{z}e^{-zt}\Big|_{t=0}^{t=b} - \int_0^b \left(-\frac{1}{z}\right)e^{-zt}\,dt$$

$$= \lim_{b\to\infty} \left[-\frac{b}{z}e^{-zb} - 0\right] + \frac{1}{z}\int_0^b e^{-zt}\,dt$$

$$= \lim_{b\to\infty} -\frac{be^{-zb}}{z} - \frac{1}{z^2}\left[e^{-zt}\Big|_{t=0}^{t=b}\right.$$

$$= \lim_{b\to\infty} -\frac{be^{-zb}}{z} - \frac{1}{z^2}\left[e^{-zb} - 1\right]$$

As long as $z > 0$, we get $\lim_{b\to\infty} e^{-zb} = 0$, so for such z, the first two terms vanish in the limit. The third term is not affected by the limit.

Thus we have obtained

$$\boxed{\mathcal{L}\{t\}(z) = \frac{1}{z^2}}$$