Quiz 15 MTH335 Fall 2025

Monday, November 17, 2025 11:32 AM

Find gen. soln for

$$\vec{x}' = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \vec{x}$$

Soln: Find e-values of $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$:

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\downarrow \text{QF}$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2}$$

$$= 1 \pm \frac{1}{2}\sqrt{-4} = 1 \pm \frac{2i}{2} = 1 \pm i$$

Find e-vec for $\lambda = 1+i$

$$A - \lambda I = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1+i & 0 \\ 0 & 1+i \end{bmatrix}$$

$$= \begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix}$$

So $-iv_1 - v_2 = 0$

$$\rightarrow v_2 = -iv_1 \Rightarrow \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -iv_1 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ -i \end{bmatrix} = v_1 \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right)$$

($\vec{a}$ = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{b}$ = $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$)

So our e-pair is $(\lambda, \vec{v}) = \left(1+i, \begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ -1 \end{bmatrix}\right)$

($\alpha = 1$, $\beta = 1$)

Thus by Thm 4.2.12 in book: gen soln is

$$\vec{x}(t) = c_1 \vec{x}_1(t) + c_2 \vec{x}_2(t),$$

where

$$\vec{x}_1(t) = e^t \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cos(t) - \begin{bmatrix} 0 \\ -1 \end{bmatrix} \sin(t) \right)$$

$$\vec{x}_2(t) = e^t \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin(t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cos(t) \right)$$