Quiz 14 MTH 335 Fall 2025

Tuesday, November 11, 2025 8:48 AM

Solve $\vec{x}' = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \vec{x}$

<u>Soln</u>: Find the e-vals of $A := \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$:

$$\lambda^2 - tr(A)\lambda + det(A) = 0$$

$$\lambda^2 - 0\lambda + (-1-0) = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$

<u>Find e-vectors</u>

<u>for $\lambda = 1$</u>

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

$0v_1 + v_2 = 0$

$v_2 = 0$

$\Rightarrow \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Rightarrow$ eigenpair

$$(\lambda, \vec{v}) = \left(1, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

<u>for $\lambda = -1$</u>

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} - (-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

$2v_1 + v_2 = 0$

$v_2 = -2v_1$

$\Rightarrow \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -2v_1 \end{bmatrix}$

$= v_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\Rightarrow$ eigenpair

$$(\lambda, \vec{v}) = \left(-1, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$$

Therefore general soln of the system is

$$\vec{x}(t) = c_1 e^{\lambda t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-\lambda t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$