# Quiz 13 MTH 335 Fall 2025

Tuesday, November 11, 2025 8:29 AM

$$\begin{cases} y''+2y'+2y=\delta(t-\pi) \\ y(0)=1,\ y'(0)=0 \end{cases}$$

Take $\mathcal{L}$ to get

$$\underbrace{\mathcal{L}\{y''+2y'+2y\}}_{\downarrow \text{ linearity of } \mathcal{L}} = \underbrace{\mathcal{L}\{\delta(t-\pi)\}}_{\downarrow \text{ table}}$$

$$\underbrace{\mathcal{L}\{y''\}+2\mathcal{L}\{y'\}+2\mathcal{L}\{y\}}_{\downarrow \text{ table}} = e^{-\pi s}$$

$$\left[s^2\mathcal{L}\{y\}-s\,y(0)-y'(0)\right]+2\left[s\mathcal{L}\{y\}-y(0)\right]+2\mathcal{L}\{y\}=e^{-\pi s}$$

↓ use initial data

$$\left[s^2\mathcal{L}\{y\}-s-0\right]+2\left[s\mathcal{L}\{y\}-1\right]+2\mathcal{L}\{y\}=e^{-\pi s}$$

$$\mathcal{L}\{y\}\left[s^2+2s+2\right]=e^{-\pi s}+s+2$$

$$\mathcal{L}\{y\}=\frac{e^{-\pi s}+s+2}{s^2+2s+2}$$

complete square to get $(s+1)^2+1$ since it doesn't factor

break up to match shift in the denominator

$$(\dagger) \qquad = \frac{e^{-\pi s}}{(s+1)^2+1}+\frac{s+1}{(s+1)^2+1}+\frac{1}{(s+1)^2+1}$$

Now recall from table:

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}=\sin(t)$$

And inverse form of 1st translation thm:

$$\mathcal{L}^{-1}\{F(s-a)\}=e^{at}f(t)$$

So we get

$$(*) \qquad \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{(s-(-1))^2+1}\right\} = e^{-t}\sin(t)$$

Similarly from $\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}=\cos(t)$ implies

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\}=\mathcal{L}^{-1}\left\{\frac{s-(-1)}{(s-(-1))^2+1}\right\} = e^{-t}\cos(t)$$

Finally inverse form of 2nd translation thm: $\mathcal{L}^{-1}\{e^{-as}F(s)\}=f(t-a)\overbrace{\mathcal{U}}^{\text{heaviside fnct}}(t-a)$

implies alongside $(*)$ that

$$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{(s+1)^2+1}\right\}=e^{-(t-\pi)}\sin(t-\pi)\,\mathcal{U}(t-\pi)$$

Therefore we take $\mathcal{L}^{-1}$ of $(\dagger)$ to get

$$y(t)=\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{(s+1)^2+1}+\frac{s+1}{(s+1)^2+1}+\frac{1}{(s+1)^2+1}\right\}$$

$$= e^{-(t-\pi)}\underbrace{\sin(t-\pi)}_{=-\sin(t)}\,\mathcal{U}(t-\pi)+e^{-t}\cos(t)+e^{-t}\sin(t)$$

if you want to further simplify