Quiz 10 MTH 335 Fall 2025

Monday, October 13, 2025 12:06 PM

We know $\mathcal{L}\{y'\}(s) = s\mathcal{L}\{y\}(s) - y(0)$

Want to show:

$$\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0)$$

<u>Soln</u>: Calculate

$$\mathcal{L}\{y''(t)\}(s) = \int_0^\infty y''(t)e^{-st}\,dt$$

$u = e^{-st}$ $\quad dv = y''(t)dt$

$du = -se^{-st}$ $\quad v = y'(t)$

$$= \lim_{b\to\infty}\int_0^b y''(t)e^{-st}\,dt$$

$$\overset{\text{(parts)}}{=} \lim_{b\to\infty}\left[e^{-st}y'(t)\Big|_{t=0}^{t=b} - \int_0^b y'(t)(-se^{-st})\,dt\right]$$

$$= \lim_{b\to\infty} \cancel{e^{-sb}y'(b)} - y'(0) + s\int_0^b y'(t)e^{-st}\,dt$$

0 as $b\to\infty$ for proper values of s

$$= -y'(0) + s\underbrace{\int_0^\infty y'(t)e^{-st}\,dt}_{=\mathcal{L}\{y'\},\text{ by def of }\mathcal{L}}$$

$\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$

$$= -y'(0) + s\left[s\mathcal{L}\{y\} - y(0)\right]$$

$$= s^2\mathcal{L}\{y\}(s) - sy(0) - y'(0),$$

as was to be shown.