

Quiz 14 MTH140 Fall 2025

Monday, November 17, 2025

11:20 AM

Solve IVP:

$$\begin{cases} y''(x) = x^2 + 3x + 2 \\ y(1) = 2, y'(1) = 3 \end{cases}$$

Soln: First take anti-derivatives to get

$$y'(x) = D^{-1}[x^2 + 3x + 2] = \frac{x^3}{3} + \frac{3}{2}x^2 + 2x + C$$

Take another to get

$$\begin{aligned} y(x) &= D^{-1}\left[\frac{x^3}{3} + \frac{3}{2}x^2 + 2x + C\right] \\ &= \frac{x^4}{12} + \frac{1}{2}x^3 + x^2 + Cx + D \end{aligned}$$

Apply given info

$$\begin{cases} 2 = \underset{\substack{\uparrow \\ \text{given}}}{y(1)} = \underset{\substack{\uparrow \\ \text{computed}}}{\frac{1}{12} + \frac{1}{2} + 1 + C + D} & (i) \\ 3 = \underset{\substack{\downarrow \\ \text{given}}}{y'(1)} = \underset{\substack{\downarrow \\ \text{computed}}}{\frac{1}{3} + \frac{3}{2} + 2 + C} & (ii) \end{cases}$$

Solve (ii) to get C:

$$3 = \frac{2}{6} + \frac{9}{6} + \frac{12}{6} + C$$

$$3 = \frac{23}{6} + C$$

$$C = 3 - \frac{23}{6} = \frac{18}{6} - \frac{23}{6} = -\frac{5}{6}$$

Plug into (i) to get

$$2 = \frac{1}{12} + \frac{6}{12} + \frac{12}{12} - \overset{\frac{10}{12}}{\underset{\substack{\uparrow \\ \text{computed}}}{\left(\frac{5}{6}\right)}} + D$$

$$\frac{24}{12} = \frac{14}{12} + D$$

$$\rightarrow D = \frac{10}{12} = \frac{5}{6}$$

Therefore soln of IVP is

$$y(x) = \frac{x^4}{12} + \frac{x^3}{3} + x^2 - \frac{5}{6}x + \frac{5}{6}$$