MTH 140 Quiz 11 Fall 2025

Tuesday, October 21, 2025 12:27 PM

Quiz 11 (due 15 Oct): The Stefan-Boltzmann law is an equation that relates the power radiated from a black body (e.g. black hole) in terms of its temperature. It is given by $J = \sigma T^4$ where J represents the total energy radiated per unit surface area of the black body per unit time, $\sigma \approx 5.670$ is a constant called the Stefan-Boltzmann constant, and T represents temperature (measured in Kelvin, K).

If the temperature is decreasing at a rate of $3\frac{\text{K}}{\text{hr}}$, then how is the total energy radiated per unit surface area per unit time changing when the temperature is $T = 100K$?

$J = \sigma T^4$

Given: $\frac{dT}{dt} = -3$

$T = 100$

We seek $\frac{dJ}{dt}$

Compute $\frac{d}{dt} J = \frac{d}{dt} \sigma T^4$

$\frac{dJ}{dt} = 4\sigma T^3 \frac{dT}{dt}$

$\Rightarrow \left.\frac{dJ}{dt}\right|_{T=100,\ \frac{dT}{dt}=-3} = 4(\sigma)(100)^3(-3) \approx -6804000$ ($\sigma \approx 5.67$)