Quiz 10 MTH 140 Fall 2025

Monday, October 13, 2025 10:53 AM

$$y^2 = \frac{x^2(1-x)}{1+x}$$

can write as $x^2 - x^3$

Take $\frac{d}{dx}$:

$$\frac{d}{dx} y^2 = \frac{d}{dx}\left[\frac{x^2-x^3}{1+x}\right]$$

$$2y\frac{dy}{dx} = \frac{(1+x)(2x-3x^2) - (x^2-x^3)(1)}{(1+x)^2}$$

$= 2x - 3x^2 + 2x^2 - 3x^3 - x^2 + x^3$

$= -2x^3 - 2x^2 + 2x$

Solve for $\frac{dy}{dx}$ to get

$$\frac{dy}{dx} = \frac{-2x^3 - 2x^2 + 2x}{y(1+x)^2}$$

Now slope at our point $\left(\frac{1}{2}, \frac{1}{\sqrt{12}}\right)$ is

$$\left.\frac{dy}{dx}\right|_{\left(\frac{1}{2}, \frac{1}{\sqrt{12}}\right)} = \frac{-2\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{12}}\right)\left(1+\frac{1}{2}\right)^2} = \frac{-\frac{1}{4} - \frac{1}{2} + 1}{\left(\frac{1}{\sqrt{12}}\right)\left(\frac{3}{2}\right)^2}$$

$\uparrow$ x $\quad \uparrow$ y

$$= \frac{\frac{1}{4}}{\frac{9}{4\sqrt{12}}} = \left(\frac{1}{\cancel{4}}\right)\left(\frac{\cancel{4}\sqrt{12}}{9}\right) = \frac{\sqrt{12}}{9}$$

Thus the tangent line is line thru $\left(\frac{1}{2}, \frac{1}{\sqrt{12}}\right)$ with slope $\frac{\sqrt{12}}{9}$:

$$y - \frac{1}{\sqrt{12}} = \frac{\sqrt{12}}{9}\left(x - \frac{1}{2}\right)$$