

Quiz 10 MTH 140 Fall 2025

Monday, October 13, 2025 10:53 AM

$y^2 = \frac{x^2(1-x)}{1+x}$ *can write as $x^2 - x^3$*

Take $\frac{d}{dx}$:

$$\frac{d}{dx} y^2 = \frac{d}{dx} \left[\frac{x^2 - x^3}{1+x} \right]$$

$$2y \frac{dy}{dx} = \frac{(1+x)(2x - 3x^2) - (x^2 - x^3)(1)}{(1+x)^2}$$

 *$= 2x - 3x^2 + 2x^2 - 3x^3 - x^2 + x^3$
 $= -2x^3 - 2x^2 + 2x$*

Solve for $\frac{dy}{dx}$ to get

$$\frac{dy}{dx} = \frac{-2x^3 - 2x^2 + 2x}{y(1+x)^2}$$

Now slope at our point $(\frac{1}{2}, \frac{1}{\sqrt{12}})$ is

$$\left. \frac{dy}{dx} \right|_{(\frac{1}{2}, \frac{1}{\sqrt{12}})} = \frac{-2(\frac{1}{2})^3 - 2(\frac{1}{2})^2 + 2(\frac{1}{2})}{(\frac{1}{\sqrt{12}})(1+\frac{1}{2})^2} = \frac{-\frac{1}{4} - \frac{1}{2} + 1}{(\frac{1}{\sqrt{12}})(\frac{3}{2})^2}$$

$$= \frac{\frac{1}{4}}{\frac{9}{4\sqrt{12}}} = \left(\frac{1}{4}\right)\left(\frac{4\sqrt{12}}{9}\right) = \frac{\sqrt{12}}{9}$$

(Note: In the original image, the 4 in the denominator and the 4 in the numerator are crossed out.)

Thus the tangent line is line thru $(\frac{1}{2}, \frac{1}{\sqrt{12}})$ with slope $\frac{\sqrt{12}}{9}$:

$$y - \frac{1}{\sqrt{12}} = \frac{\sqrt{12}}{9} \left(x - \frac{1}{2}\right)$$

