MTH 452 Quiz 6

Sunday, March 31, 2024 5:00 PM

$R = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$

$N = \{(0,0,n) : n \in \mathbb{Z}\}$

Show N is an ideal:

<u>closed under $+$:</u>

$(0,0,n) + (0,0,m) = (0,0,n+m)$

$n+m \in \mathbb{Z}$, so $(0,0,n+m) \in N$

$\Rightarrow$ closed under $+$

<u>absorbs mult</u>

for any $(a,b,c) \in R$,

$(a,b,c)N = \{(a,b,c)(0,0,n) : n \in \mathbb{Z}\}$

$= \{(0,0,cn) : n \in \mathbb{Z}\} \subseteq N$

$(cn \in \mathbb{Z})$

Thus N is an ideal of R.

Now

$R/N = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} / N$

Now

$$R/N = \mathbb{Z}\times\mathbb{Z}\times\mathbb{Z}/N$$

elements of R/N look like

$$\{(a,b,c)+N : (a,b,c)\in R\}$$

$$\{(a,b,c+n) : (a,b,c)\in R,\ n\in\mathbb{Z}\}$$

these two elements completely determine what element of R/N $(a,b,c)+N$ is

can be completely determined by choice of n

$$\Downarrow$$

R/N isomorphic to $\mathbb{Z}\times\mathbb{Z}$