# Quiz 9 MTH 427/527 Fall 2024

Wednesday, October 2, 2024 9:33 AM

Prove that for $k \geq m$, if $\sum_{i=m}^{\infty} a_i$ converges, then $\sum_{i=k}^{\infty} a_i$ converges.

<u>Proof</u>: Assume $\sum_{i=m}^{\infty} a_i$ converges. This means the partial sum $s_n = \sum_{i=m}^{n} a_i$, $n \geq m$ converges to a limit $S$, so we know $\forall \epsilon > 0\ \exists N\ \forall n > N\ |s_n - S| < \epsilon$.

For any $n \geq k$, compute

$$s_n = \sum_{i=m}^{n} a_i = a_m + a_{m+1} + \dots + a_{k-1} + \sum_{i=k}^{n} a_i$$

Therefore, the partial sums $\sigma_n$ are defined by

$$\sigma_n = \sum_{i=k}^{n} a_i = s_n - a_m - a_{m+1} - a_{m+2} - \dots - a_{k-1}$$

We claim that $\lim_{n \to \infty} \sigma_n = S - a_m - a_{m+1} - \dots - a_{k-1}$.

To prove it, let $\epsilon > 0$ and choose $N$ so that $\forall n > N$, $|s_n - S| < \epsilon$.

Now compute

$$|\sigma_n - (S - a_m - a_{m+1} - \dots - a_{k-1})| = |(s_n - \cancel{a_m} - \cancel{a_{m+1}} - \dots - \cancel{a_{k-1}}) - (S - \cancel{a_m} - \cancel{a_{m+1}} - \dots - \cancel{a_{k-1}})|$$

$$\overset{\text{algebra}}{=} |s_n - S| < \epsilon,$$

Completing the proof. ∎