Quiz 8 MTH 427/527 Fall 2024

Wednesday, October 2, 2024 9:26 AM

Show $a_n = \frac{(-1)^n}{n}$ is a Cauchy sequence.

Need to show $\forall \epsilon > 0\ \exists N\ \forall n, m > N$, $|a_n - a_m| < \epsilon$ (i.e. $\left|\frac{(-1)^n}{n} - \frac{(-1)^m}{m}\right| < \epsilon$)

Thinking: assume $m \geq n$, then $\frac{1}{n} \leq \frac{1}{m}$

$$\left|\frac{(-1)^n}{n} - \frac{(-1)^m}{m}\right| \underset{\Delta\text{-ineq}}{\leq} \frac{1}{n} + \frac{1}{m} \leq \frac{2}{m} \overset{\text{need}}{<} \epsilon$$

$$\frac{m}{2} > \frac{1}{\epsilon}$$

$$m > \frac{2}{\epsilon}$$

Proof: Let $\epsilon > 0$. Choose $N > \frac{2}{\epsilon}$. Let $m, n > N$ with $m \geq n$.

Then, $\frac{1}{n} \leq \frac{1}{m}$ and so in particular, $m > N > \frac{2}{\epsilon} \Rightarrow \frac{2}{m} < \epsilon$.

Therefore, compute

$$|a_n - a_m| = \left|\frac{(-1)^n}{n} - \frac{(-1)^m}{m}\right| \overset{\Delta\text{-ineq}}{\leq} \frac{1}{n} + \frac{1}{m} \leq \frac{2}{m} < \epsilon,$$

completing the proof. ∎