

Quiz 7 MTH 427/527 Fall 2024

Wednesday, October 2, 2024

9:22 AM

Find $\limsup_{n \rightarrow \infty} \frac{1}{n}$ and $\liminf_{n \rightarrow \infty} \frac{1}{n}$.

Consider the numbers

$$u_i = \sup \left\{ \frac{1}{j} : j \geq i \right\} = \sup \left\{ \frac{1}{i}, \frac{1}{i+1}, \frac{1}{i+2}, \dots \right\} = \frac{1}{i}$$

and

$$l_i = \inf \left\{ \frac{1}{j} : j \geq i \right\} = \inf \left\{ \frac{1}{i}, \frac{1}{i+1}, \dots \right\} = 0$$

Now by Definition 2.1.5,

$$\limsup_{n \rightarrow \infty} \frac{1}{n} = \inf \{ u_1, u_2, u_3, \dots \} = \inf \left\{ 1, \frac{1}{2}, \frac{1}{3}, \dots \right\} = 0$$

and by Definition 2.1.6,

$$\liminf_{n \rightarrow \infty} \frac{1}{n} = \sup \{ l_1, l_2, \dots \} = \sup \{ 0 \} = 0$$