Quiz 4 MTH 427/527 Fall 2024

Theorem: Let $a \in \mathbb{Q}$. Prove that it is not possible for both $a = 0$ and $a > 0$.
Proof: Since $a \in \mathbb{Q}$ and $a > 0$, there exist integers $p, q \in \mathbb{Z}^+$ so that $a = [(p, q)]$. Since $a = 0$, there exists an integer $t \in \mathbb{Z}$ so that $a = [(0, t)]$. This means that

$$[(p, q)] = a = [(0, t)].$$

But this is a contradiction because these two equivalence classes cannot be equal as sets: for instance, $(0, 2) \in [(0, t)]$ while $(0, 2) \notin [(p, q)]$ because $0 \notin \mathbb{Z}^+$. ■