

# MTH 427/527 Quiz 3 Fall 2024

Show  $a(b+c) = ab+ac$

Proof: let  $a, b, c \in \mathbb{Q}$  and write  $a = \frac{p_1}{q_1}$ ,  $b = \frac{p_2}{q_2}$ , and  $c = \frac{p_3}{q_3}$ .

Then

$$a(b+c) = \frac{p_1}{q_1} \left( \frac{p_2}{q_2} + \frac{p_3}{q_3} \right)$$

$$\stackrel{(1.3.6)}{=} \frac{p_1}{q_1} \left( \frac{p_2 q_3 + p_3 q_2}{q_2 q_3} \right)$$

$$(*) \rightarrow \stackrel{(1.3.7)}{=} \frac{p_1 p_2 q_3 + p_1 p_3 q_2}{q_1 q_2 q_3}$$

not equal yet!

On the other hand,

$$ab+ac = \left( \frac{p_1}{q_1} \right) \left( \frac{p_2}{q_2} \right) + \left( \frac{p_1}{q_1} \right) \left( \frac{p_3}{q_3} \right)$$

$$\stackrel{(1.3.7)}{=} \frac{p_1 p_2}{q_1 q_2} + \frac{p_1 p_3}{q_1 q_3} \stackrel{(1.3.6)}{=} \frac{p_1 p_2 q_1 q_3 + p_1 p_3 q_1 q_2}{q_1^2 q_2 q_3}$$

But

$$a(b+c) = a(b+c)(1)$$

$$= a(b+c) \left( \frac{q_1}{q_1} \right)$$

$$\stackrel{(*)}{=} \left( \frac{p_1 p_2 q_3 + p_1 p_3 q_2}{q_1 q_2 q_3} \right) \left( \frac{q_1}{q_1} \right)$$

$$\stackrel{(1.3.7)}{=} \frac{p_1 p_2 q_3 q_1 + p_1 p_3 q_2 q_1}{q_1^2 q_2 q_3}$$

now equal!

Completing the proof. ▀