MTH 427 Quiz 2 Fall 2024

Show $m \sim_R n$ iff $m-n$ is even is (transitive).

$a \sim_R b$ and $b \sim_R c$

$\Downarrow$

$a \sim_R c$

Proof: Assume that $a \sim_R b$ and $b \sim_R c$.

Then $a-b$ and $b-c$ are even, so there exist integers $k_1, k_2 \in \mathbb{Z}$ so that

$a-b = 2k_1$ and $b-c = 2k_2$.

Now consider

$$\begin{aligned} a-c &= a-c+b-b \\ &= (a-b)+(b-c) \\ &= 2k_1+2k_2 \\ &= 2(k_1+k_2) \end{aligned}$$

This shows $a-c$ is even, thus $a \sim_R c$, completing the proof. ▨