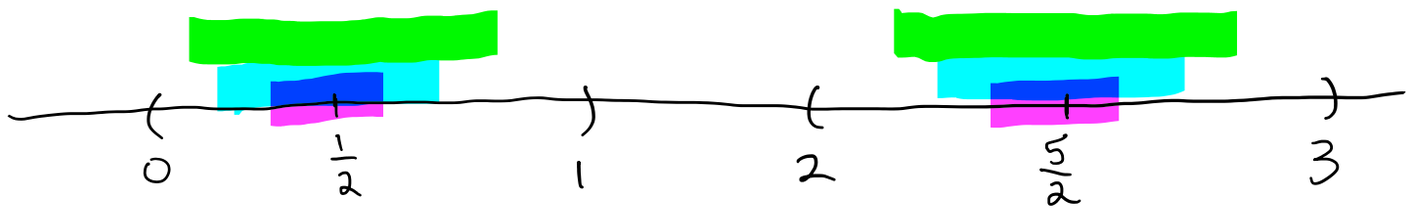
Quiz 13 MTH 427/527 Fall 2024

Thursday, November 7, 2024 8:19 AM

Find open cover of $(0,1) \cup (2,3)$ without finite subcover



notice that sequence $\frac{n}{2n+1} \xrightarrow{n\to\infty} \frac{1}{2}$, so consider open sets

$$I_n = \left(\tfrac{1}{2} - \tfrac{n}{2n+1}, \tfrac{1}{2} + \tfrac{n}{2n+1}\right) \cup \left(\tfrac{5}{2} - \tfrac{n}{2n+1}, \tfrac{5}{2} + \tfrac{n}{2n+1}\right)$$

$$I_1 = \left(\tfrac{1}{2} - \tfrac{1}{3}, \tfrac{1}{2} + \tfrac{1}{3}\right) \cup \left(\tfrac{5}{2} - \tfrac{1}{3}, \tfrac{5}{2} + \tfrac{1}{3}\right)$$

$$I_2 = \left(\tfrac{1}{2} - \tfrac{2}{5}, \tfrac{1}{2} + \tfrac{2}{5}\right) \cup \left(\tfrac{5}{2} - \tfrac{2}{5}, \tfrac{5}{2} + \tfrac{2}{5}\right)$$

$$I_3 = \left(\tfrac{1}{2} - \tfrac{3}{7}, \tfrac{1}{2} + \tfrac{3}{7}\right) \cup \left(\tfrac{5}{2} - \tfrac{3}{7}, \tfrac{5}{2} + \tfrac{3}{7}\right)$$

Since $\bigcup_{n=1}^{\infty} I_n = (0,1) \cup (2,3)$, it is an open cover. But no finite subcover exists because it takes all of the infinite open sets to cover all the points (do you see why?)