Quiz 12 MTH 427/527 Fall 2024

Thursday, November 7, 2024 8:11 AM

Let $I_n = \left[1 - \frac{n}{n+1}, 1 + \frac{n}{n+1}\right]$ for $n = 1, 2, 3, \ldots$

$I_1 = \left[1 - \frac{1}{2}, 1 + \frac{1}{2}\right] = \left[\frac{1}{2}, \frac{3}{2}\right]$

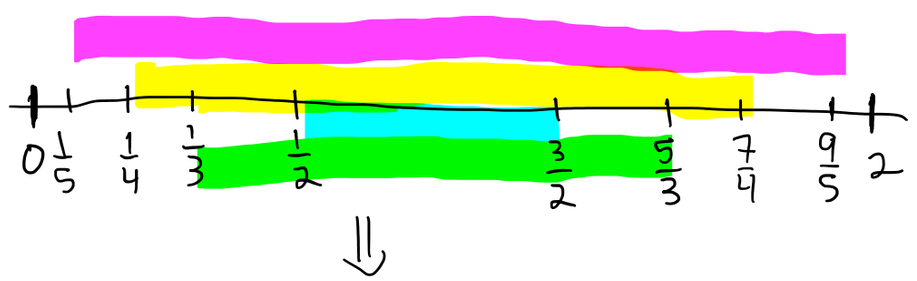
$I_2 = \left[1 - \frac{2}{3}, 1 + \frac{2}{3}\right] = \left[\frac{1}{3}, \frac{5}{3}\right]$

$I_3 = \left[1 - \frac{3}{4}, 1 + \frac{3}{4}\right] = \left[\frac{1}{4}, \frac{7}{4}\right]$

$I_4 = \left[1 - \frac{4}{5}, 1 + \frac{4}{5}\right] = \left[\frac{1}{5}, \frac{9}{5}\right]$

$1 - \frac{n}{n+1} \xrightarrow{n\to\infty} 1 - 1 = 0$

$1 + \frac{n}{n+1} \xrightarrow{n\to\infty} 1 + 1 = 2$



Open, closed, or neither?:

(a) $\bigcap_{n=1}^{\infty} I_n = \bigcap_{n=1}^{\infty} \left[1 - \frac{n}{n+1}, 1 + \frac{n}{n+1}\right]$

$= \left[\frac{1}{2}, \frac{3}{2}\right]$

closed!

(b) $\bigcup_{n=1}^{\infty} I_n = \bigcup_{n=1}^{\infty} \left[1 - \frac{n}{n+1}, 1 + \frac{n}{n+1}\right]$

$= (0, 1)$

open!

endpoints not included because they do not appear in any of the I_n