HW3 MTH 300 Fall 2024

Sunday, September 8, 2024 4:49 PM

§ 2.3

#1) "If a matrix has nonzero determinant, then it is an invertible matrix."

#4) "If a function is a polynomial, then it is a rational function."

§ 2.5

#5) $(P \wedge (\sim P)) \vee Q$

P	Q	$\sim P$	$P \wedge (\sim P)$	$(P \wedge (\sim P)) \vee Q$
T	T	F	F	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	F

#7) $(P \wedge (\sim P)) \rightarrow Q$

P	Q	$P \wedge (\sim P)$	$(P \wedge (\sim P)) \rightarrow Q$
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T

§ 2.6

#9) are $P \wedge Q$ and $\sim(\sim P \vee \sim Q)$ equivalent?

P	Q	$P \wedge Q$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$\sim(\sim P \vee \sim Q)$
T	T	T	F	F	F	T
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F	F	T	T	T	F

same, so they are equivalent

#11) $(\sim P) \wedge (P \rightarrow Q)$ equivalent to $\sim(Q \rightarrow P)$?

P	Q	$\sim P$	$P \rightarrow Q$	$(\sim P) \wedge (P \rightarrow Q)$	$Q \rightarrow P$	$\sim(Q \rightarrow P)$
T	T	F	T	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	F	T	T	T	T	F

not same, so not equivalent

§ 2.7 2,3

#2) $\forall x \in \mathbb{R}\ \exists n \in \mathbb{N}\quad x^n \geq 0$

"for every real number x, there exists a natural number n such that $x^n \geq 0$"

#3) $\exists a \in \mathbb{R}\ \forall x \in \mathbb{R}\quad ax = x$

"there exists a real number a such that for all real numbers x, $ax = x$"

§ 2.9

#1) "If f is a polynomial and its degree is > 2, then f' is not constant"

Let $\mathcal{P}$ = polynomials, $\mathcal{C}$ = constant functions

$$(\deg(f) = n \wedge n > 2) \rightarrow \sim(f' \in \mathcal{C})$$

#2) "The number x is positive, but the number y is not positive."

$$(x > 0) \wedge \sim(y > 0)$$

§ 2.10

#9)
$$\sim\left[\exists a \in \mathbb{R}\ \forall x \in \mathbb{R}\ (a + x = x)\right]$$
$$= \forall a \in \mathbb{R}\ \sim(\forall x \in \mathbb{R}\ (a + x = x))$$
$$= \forall a \in \mathbb{R}\ \exists x \in \mathbb{R}\ (\sim(a + x = x))$$
$$= \forall a \in \mathbb{R}\ \exists x \in \mathbb{R}\ (a + x \neq x)$$

book, p.61

$\sim(P \rightarrow Q) = P \wedge (\sim Q)$

#10)
$$\sim\left[(\deg(f) = n \wedge n > 2) \rightarrow \sim(f' \in \mathcal{C})\right]$$
$$= (\deg(f) = n \wedge n > 2) \wedge \sim(\sim(f' \in \mathcal{C}))$$

$\sim(\sim P) = P$

$$= (\deg(f) = n \wedge n > 2) \wedge (f' \in \mathcal{C})$$