HW14 MTH 300 Fall 2024

Section 12.1 #2 Suppose $A = \{a, b, c, d\}$, $B = \{2, 3, 4, 5, 6\}$, and $f = \{(a, 2), (b, 3), (c, 4), (d, 5)\}$. State the domain and range of f. Find $f(b)$ and $f(d)$.
Solution: Here, the domain is the set of first coordinates that appear, which is

$$\text{dom}(f) = \{a, b, c, d\} = A.$$

The range is the set of second coordinates that appear, which is

$$\text{range}(f) = \{2, 3, 4, 5\} \subset B.$$

Since $(b, 3) \in f$, we write $f(b) = 3$ and since $(d, 5) \in f$, we write $f(d) = 5$.

Section 12.2 #2 Consider the logarithm function $\ln: (0, \infty) \to \mathbb{R}$. Decide whether this function is injective and whether it is surjective.
Solution: The graph of ln can be seen at this link: `https://www.desmos.com/calculator/0hn68famn0`. The function is injective because if $ln(x) = ln(y)$, then plugging both sides into the function e^x gives $x = y$. The function is surjective because for any $y \in \mathbb{R}$, we know that $e^y \in (0, \infty)$ and so we see $\ln(e^y) = y$, showing that every $y \in (0, \infty)$ is mapped to by some element of $(0, \infty)$.

Section 12.2 #4: A function $f: \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ is defined as $f(n) = (2n, n + 3)$. Verify whether this function is injective and whether it is surjective.
Solution: see notes from the 2024.12.05 review day where this was proven.

Section 12.5 #4 The function $f: \mathbb{R} \to (0, \infty)$ is defined by $f(x) = e^{x^3+1}$ is bjiective. Find its inverse.
Solution: From $y = e^{x^3+1}$. Taking ln of both sides gives $\ln(y) = x^3 + 1$, subtract 1 and then take a cube root to get $x = \sqrt[3]{\ln(y) - 1}$. Thus we have $f^{-1}(x) = \sqrt[3]{\ln(x) - 1}$.

Section 12.6 #5 Consider $f: A \to B$ and $X \subset A$. Prove that $X \subset f^{-1}(f(X))$.
Proof: Let $w \in X$. So, $f(w) \in f(X)$. From this we see that $w \in f^{-1}(f(X))$, completing the proof that $X \subset f^{-1}(f(X))$. ■

Section 12.6 #7 Given a function $f: A \to B$ and $W, X \subset A$. Prove that $f(W \cap X) \subset f(W) \cap f(X)$.
Proof: Let $z \in f(W \cap X)$, meaning that there exists some $y \in W \cap X$ so that $f(y) = z$. Since $y \in W$, $f(y) \in f(W)$ aand since $y \in X$, $f(y) \in f(X)$. Thus $y \in f(W) \cap f(X)$, completing the proof. ■