# Quiz 5 MTH 450/550 Fall 2023

Friday, October 13, 2023 10:03 PM

$\pi\mathbb{Z} = \{\pi n : n \in \mathbb{Z}\}$ makes a subgroup of $\langle \mathbb{Z}, + \rangle$

To show it is a subgroup, just have to show that (by Thm 5.14):

1. closed under operation
2. identity element is in there
3. everything in there has an inverse

So consider 1:

Let $t, s \in \pi\mathbb{Z}$. Then $\exists\, n_1, n_2 \in \mathbb{Z}$ so that $t = \pi n_1$ and $s = \pi n_2$

Thus, $t + s = \pi n_1 + \pi n_2 = \pi(n_1 + n_2)$. Since $n_1 + n_2 \in \mathbb{Z}$, we conclude that $t + s \in \pi\mathbb{Z}$, so it is closed under the operation $+$. ✓

Now 2:

$0 \in \mathbb{Z}$ is the identity of $\langle \mathbb{Z}, + \rangle$

But $0 \in \pi\mathbb{Z}$ as well because $0 = 0\pi$ ✓

Now 3:

Let $t \in \pi\mathbb{Z}$ with $t = n_1\pi$. Then $-t = -n_1\pi$ is also in $\pi\mathbb{Z}$ because if $n_1 \in \mathbb{Z}$, then $-n_1 \in \mathbb{Z}$.

Now, $t^{-1} = -t$ because $t + (-t) = 0$. ✓

Thus by application of Thm 5.14, $\langle \pi\mathbb{Z}, + \rangle$ is a group.