

Quiz 3 MATH 450-550 Fall 2023

$$\langle \mathbb{R}^+, * \rangle, a * b := \sqrt{ab}$$

Check group axioms:

(associative): $a * (b * c) = a * \sqrt{bc} = \sqrt{a\sqrt{bc}}$ } not equal!

while

$$(a * b) * c = \sqrt{ab} * c = \sqrt{\sqrt{ab}c}$$

\therefore not associative

(identity): need x so that for all $a \in \mathbb{R}^+$,

$$a * x = a$$

$$\sqrt{ax} = a$$

$$ax = a^2 \quad a \neq 0$$

$$x = a$$

But this won't work for all values...

\Rightarrow no identity in this structure

(inverses): can't exist because there's no identity!

So this structure fails all group axioms!