

Written HW17 – MATH 2510 Spring 2023

In this homework, you will provide interpretations in various “finite geometries”. Your interpretations can (and probably should) be pictures of lines and points, as described by the axioms. The axioms here are not written as first-order logic formulas to make them easier to understand.

Consider the theory of four-point geometry defined by the following axioms:

- Axiom 1** There exist exactly four points.  
**Axiom 2** Each two distinct points have exactly one line that contains both of them.  
**Axiom 3** Each line is exactly on two points.

1. Draw a picture (“interpretation”) of four-point geometry.
2. Show that the axioms of four-point geometry are all independent from each other by finding an interpretation that makes Axioms 1 and 2 true but **NOT** Axiom 3; an interpretation that makes Axioms 1 and 3 true but **NOT** Axiom 2; and an interpretation that makes Axioms 2 and 3 true but **NOT** Axiom 1.

Consider six-line geometry defined by the following axioms:

- Axiom 1** There exist exactly six lines.  
**Axiom 2** Each two distinct lines have exactly one point on both of them.  
**Axiom 3** Each point is on exactly two lines.

3. Draw a picture of six-line geometry.
4. Show that the axioms of six-line geometry are all independent from each other (same way as was done for Problem 2 above).