Written HW16 – MATH 2510 Sping 2023

Consider "differential algebra theory" which has predicates ∂, $\cdot$, and $+$ which obeys the following two axioms:

$$\forall x \forall y \Big(\partial(x \cdot y) = (\partial x) \cdot y + x \cdot (\partial y) \tag{1}$$

$$\forall x \forall y \Big(\partial(x + y) = \partial(x) + \partial(y) \Big) \tag{2}$$

Write a proof of the sentence

$$\forall x \forall y \forall z \Bigg(\partial((x \cdot y) \cdot z) = \Big((\partial x) \cdot y + x \cdot (\partial y) \Big) \cdot z + (x \cdot y) \cdot (\partial z) \Bigg).$$