Written HW13 – MATH 3503 Fall 2022

Stokes theorem says, where C is the boundary curve of the surface S,

$$\iint_S \mathrm{curl}\vec{F} \cdot \mathrm{d}\vec{S} = \int_C \vec{F} \cdot \mathrm{d}\vec{r}.$$

Divergence theorem says, where S is the boundary surface of the region E in space,

$$\iiint_E \mathrm{divF}\mathrm{d}V = \iint_S \vec{F} \cdot \mathrm{d}\vec{S}.$$

In the problems below you will verify these theorems by computing both sides.

1. Compute both sides of Stokes theorem in the following scenario: $\vec{F} = \langle y^2, x, z^2 \rangle$ where S is the part of the paraboloid $z = x^2 + y^2$ lying in the first octant (i.e. when $x > 0, y > 0, z > 0$) and below the plane $z = 1$, oriented upward.

2. Compute both sides of the divergence theorem in the following scenario: $\vec{F} = \langle x^2, xy, z \rangle$ where E is the solid bounded by hte paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.