Written HW12 – MATH 3503 Fall 2022

Recall that if a surface S is parametrized by $\vec{r}(u,v)$, with $(u,v) \in D$ then the scalar surface integral is given by

$$\iint_S f(x,y,z)\mathrm{d}S = \iint_D f(\vec{r}(u,v))\,\|\vec{r}_u \times \vec{r}_v\|\,\mathrm{d}A.$$

Similarly, if S is orientiable with unit normal vector $\vec{n} = \dfrac{\vec{r}_u \times \vec{r}_v}{\|\vec{r}_u \times \vec{r}_v\|}$ and $\vec{F}$ is a vector field over S, then the flux integral over S is given by

$$\iint_S \vec{F}(x,y,z))\mathrm{d}S = \iint_D \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v)\,\mathrm{d}A.$$

Note sometimes $\vec{r}_v \times \vec{r}_u$ will be needed instead of $\vec{r}_u \times \vec{r}_v$ in order to have the correct orientation. Recall that cross products are anti-commutative, i.e. for all vectors $\vec{x}$ and $\vec{y}$, $\vec{x} \times \vec{y} = -\vec{y} \times \vec{x}$.

In the following problems, set up **but do not evaluate** the surface integral as a double integral. Include in your submission a picture of the surface and at least one normal vector to the surface.

1. $\displaystyle\iint_S x^2z^2\mathrm{d}S$ where S is the surface $x = y + 2z^2$ and $0 \leq y \leq 1$ and $0 \leq z \leq 1$.
2. $\displaystyle\iint_S \langle x, -z, y\rangle\,\mathrm{d}S$ where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ lying in the first octant (i.e. where $x > 0$, $y > 0$, and $z > 0$) and orientation toward the origin.
3. $\displaystyle\iint_S \langle x, y, z^4\rangle\mathrm{d}S$ where S is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ and upward orientation