

Written HW11 – MATH 3503 Fall 2022

Recall that Green's theorem says that if D is a simply-connected region, its boundary C is parametrized with positive orientation (counter-clockwise), and $\vec{F} = \langle P, Q \rangle$, then

$$\int_C \vec{F} \cdot d\vec{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA.$$

Green's theorem allows for the computation of a double integral as a line integral AND the computation of a line integral as a double integral. Below you will see examples of both.

1. Set up **but do not evaluate** both sides of Green's theorem for the following scenario: D is the region bounded by the graphs of $y = x$ and $y = \sqrt{x}$ and $\vec{F} = \langle xe^y, e^x \rangle$.
2. Use Green's theorem to calculate the line integral

$$\int_C (y - x)dx + (2x - y)dy,$$

where C is the boundary curve of the region lying between the graphs of $y = x$ and $y = x^2 - 2x$

3. We will use Green's theorem to calculate the double integral

$$\iint_D y - x dA,$$

where D is the unit disk (so its boundary is the unit circle). (*hint: the hard part is finding an \vec{F} that works! in other words, you need to pick P and Q so that $\vec{F} = \langle P, Q \rangle$ and $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = y - x$, but also allows you to calculate the line integral...keep it simple!*)