Written HW3 – MATH 3504 Spring 2021
Due by 1 February for timely completion credit

In this homework you will investigate and solve two popular population growth models. The first model is the logistic growth model

$$x' = rx\left(1 - \frac{x}{K}\right), \quad r > 0, K > 0, x > 0.$$

The second model is the Gompertz growth model

$$x' = rx\log\left(\frac{K}{x}\right), \quad r > 0, K > 0, x > 0.$$

In both models we do not consider negative population. Notice that both differential equations have two parameters: r (the growth rate) and K the so-called "carrying capacity".

1. Show that $x(t) = K$ is a solution of both equations. This is called the equilibrium solution.
2. Both equations are autonomous, meaning there is no dependence directly on the independent variable t on the right-hand side. Draw a sketch like those from the 20 January notes that show what happens to solutions that start above and below the carrying capacity K (don't let x start negative! We assume that $x > 0$). Make sure to include verifications at test points for your sketch.
3. Both equations are separable. Find the general solution of both differential equations. (*hint: for the logistic, you should recall the method of partial fractions from calculus 2; a simple u-substitution will integrate Gompertz*).
4. Suppose that $r = 1$ and $K = 2$ and solve each equation equipped with initial condition $x(0) = 0.5$.
5. Plot both of the solutions you find on the same graph (desmos makes this easy) and describe the visual differences you notice in each solution.