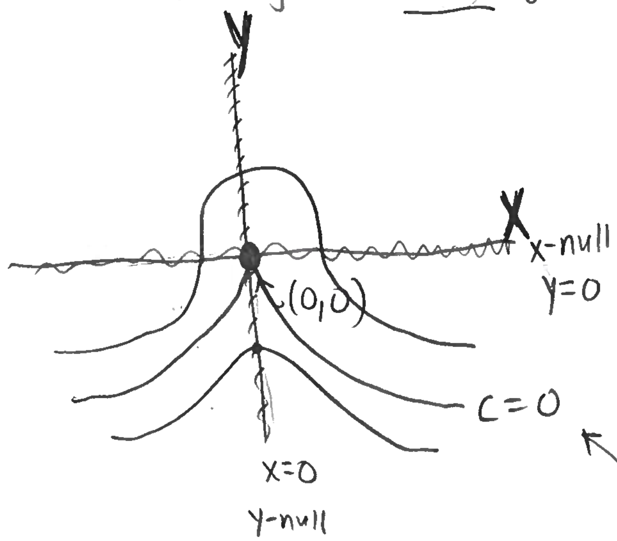
Ex: $\begin{cases} x' = y^2 \leftarrow f \\ y' = -\frac{2}{3}x \leftarrow g \end{cases}$

x-null; $x' = 0 \rightarrow 0 = y^2$, $\boxed{y = 0}$

y-null: $y' = 0 \rightarrow 0 = -\frac{2}{3}x$, $\boxed{x = 0}$

not seen before →



← Desmos

$f_x = 0 \quad f_y = 2y$

$g_x = -\frac{2}{3} \quad g_y = 0$

So, Jacobian is

$$J(0,0) = \begin{pmatrix} 0 & 0 \\ -2/3 & 0 \end{pmatrix}$$

$\det J = 0 \Rightarrow$ we can't use linearization

What to do?

Divide the two DE's:

$$\frac{x'}{y'} = \frac{y^2}{(-2/3)x} \longrightarrow \frac{dx/dt}{dy/dt} = \frac{dx}{dy} = \frac{y^2}{(-\frac{2}{3})x}$$

Separate vars: $\int -\frac{2}{3}x \, dx = \int y^2 \, dy$

$$-\frac{2}{3}\frac{x^2}{2} + C = \frac{y^3}{3} \longrightarrow \boxed{y = \sqrt[3]{\tilde{C} - x^2}} \; ; \tilde{C} = 3C$$

p.257

(#1a) Find crit pts, classify w/ Jacobian, sketch nullclines, phase diag, dir. field

$$\begin{cases} x' = x - y & \leftarrow f \\ y' = 2x(-1+y) & \leftarrow g \end{cases}$$

x-null: $x' = 0 \rightarrow 0 = x - y \rightarrow \boxed{y = x}$

y-null: $y' = 0 \rightarrow 0 = 2x(-1+y) \rightarrow x = 0, y = 1$

$f_x = 1 \quad f_y = -1$

$g_x = 2(-1+y) \quad g_y = 2x$

J(0,0)

$J(0,0) = \begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}$ evals $\rightsquigarrow$ $\det(J - \lambda I) = 0$

$\det \begin{pmatrix} 1-\lambda & -1 \\ -2 & -\lambda \end{pmatrix} = 0$

$(-\lambda)(1-\lambda) - 2 = 0$

$\lambda^2 - \lambda - 2 = 0 \rightarrow (\lambda-2)(\lambda+1) = 0$

$\lambda = 2, -1$

unstable saddle

$\det J = 0 - 2 < 0$

↓

saddle

J(1,1)

$J(1,1) = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ → could have found evals $\lambda = 1, 2$

$\det J = 2 > 0$

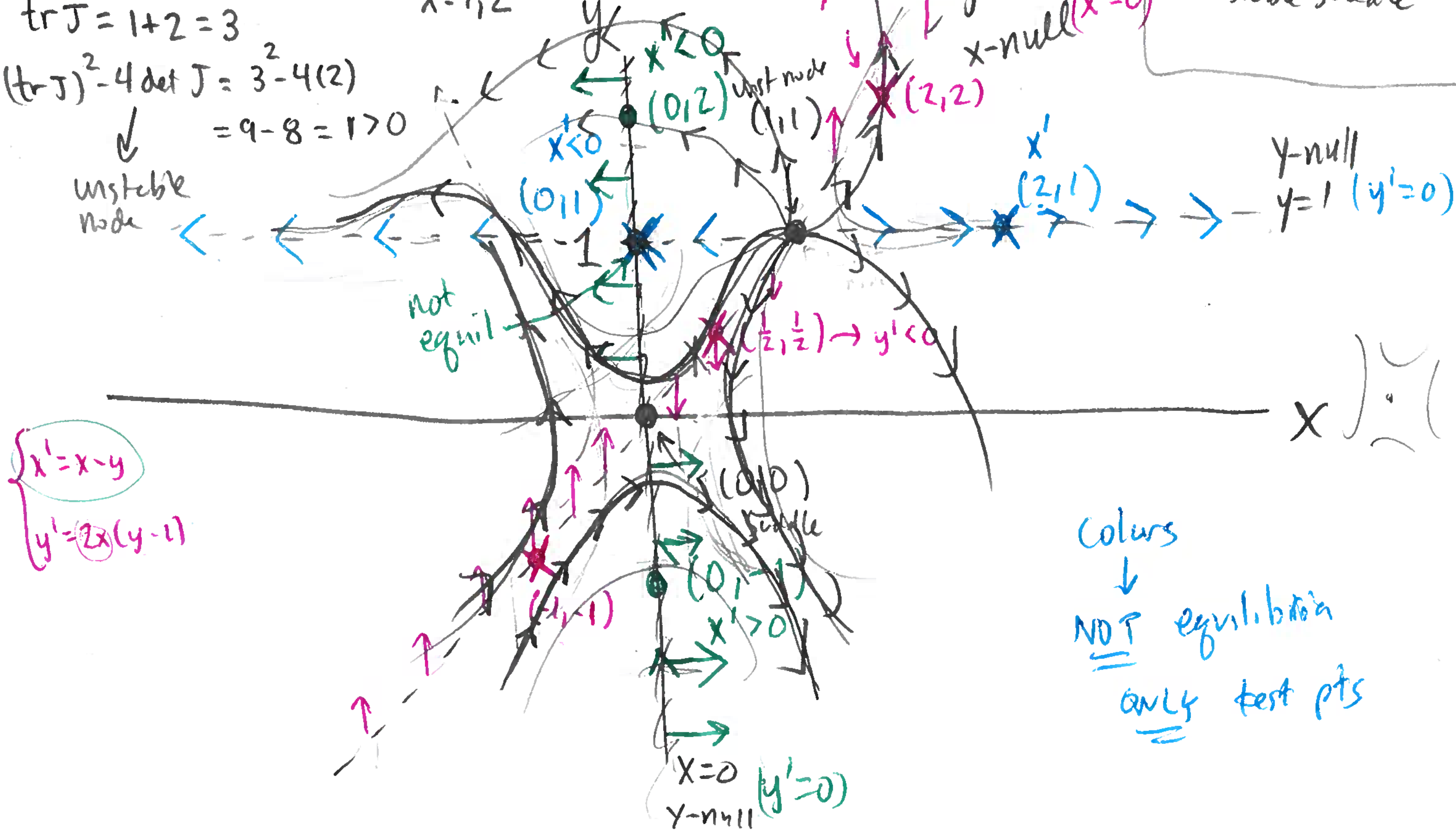
$\operatorname{tr} J = 1 + 2 = 3$

$(\operatorname{tr} J)^2 - 4 \det J = 3^2 - 4(2)$

$= 9 - 8 = 1 > 0$

↓

unstable node



$(r, a, m, b > 0)$

Lotka-Volterra

(predator-prey)

$x \sim$ prey

$y \sim$ predator

$$\begin{cases} x' = rx - axy \\ y' = -my + bxy \end{cases}$$

↑ interactions

ex) if you had 5 predators + 20 prey, then

↓

$5 \cdot 20 = 100$ interactions

- $b \sim$ conversion efficiency ~ when predator eats prey, how efficient can it be turned into higher population
- $a \sim$ fraction of prey consumed

Assumptions

* no predators $(y=0)$

↓

prey grow exponentially

$x' = rx$

* no prey $(x=0)$

↓

Predator pop decr. exp

$y' = -my$