

From last time:

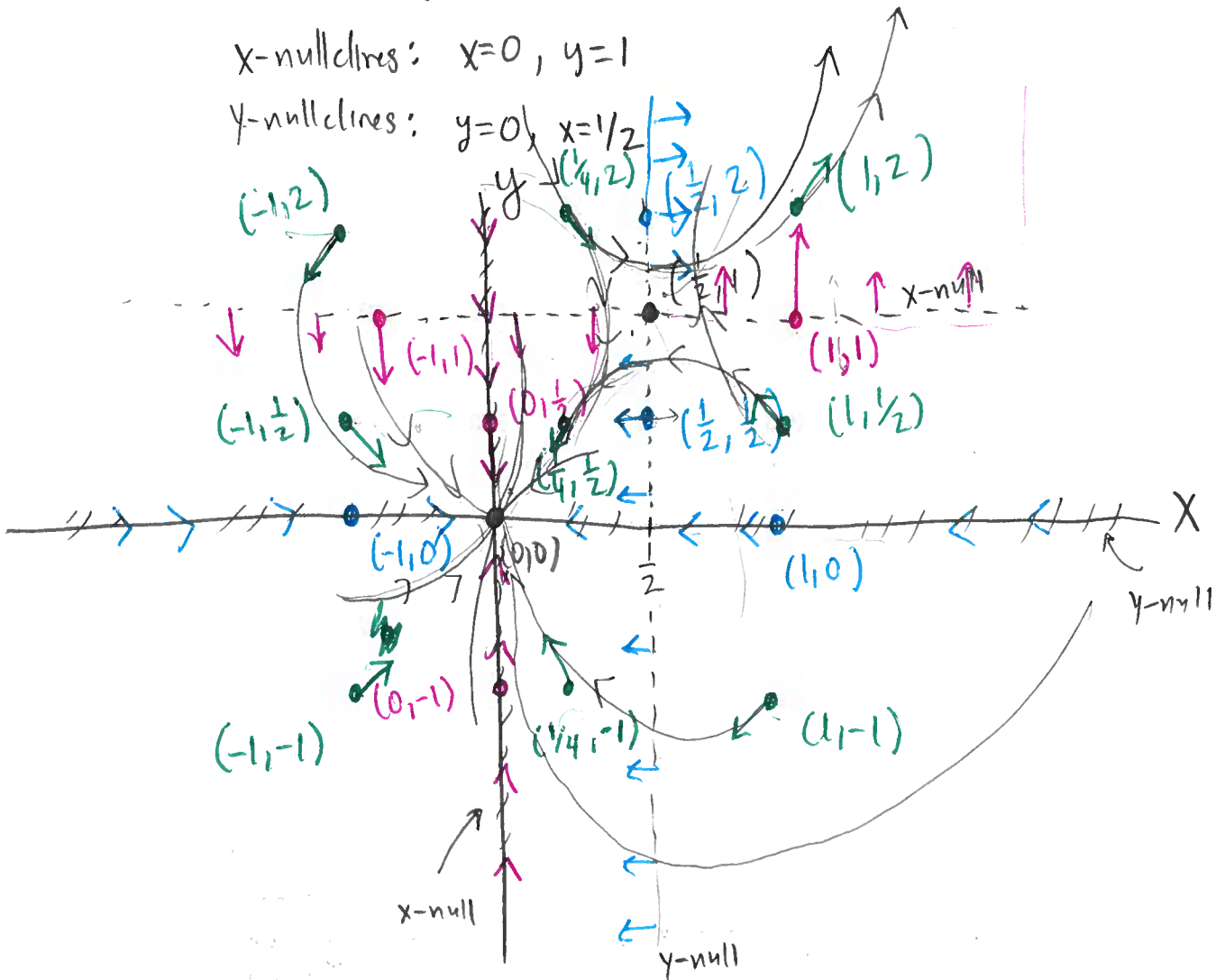
$$\begin{cases} x' = -x + xy \\ y' = -4y + 8xy \end{cases}$$

(1)

equilibria:  $(0, 0)$   $(\frac{1}{2}, 1)$   
↓ ↓  
stable node saddle

x-nullclines:  $x=0, y=1$

y-nullclines:  $y=0, x=1/2$



(2)

$$\begin{cases} x' = -x + xy \\ y' = -4y + 8xy \end{cases}$$

at  $(1, 1)$

$$x' = -1 + 1 = 0$$

$$y' = -4 + 8 = 4 > 0$$

at  $(-1, 1)$

$$x' = 0 \quad y' = -4 - 8 < 0$$

at  $(0, \frac{1}{2})$

$$x' = 0 \quad y' = -2 < 0$$

at  $(0, -1)$

$$x' = 0 \quad y' = 4 > 0$$

at  $(-1, 0)$

$$x' = 1 > 0 \quad y' = 0$$

at  $(1, 0)$

$$x' = -1 < 0, \quad y' = 0$$

at  $(\frac{1}{2}, \frac{1}{2})$

$$x' = -\frac{1}{2} + \frac{1}{4} < 0 \quad y' = 0$$

at  $(\frac{1}{2}, 2)$

$$x' = -\frac{1}{2} + 8 > 0 \quad y' = 0$$

at  $(-1, 2)$

$$x' < 0, \quad y' < 0$$

at  $(-1, \frac{1}{2})$

$$x' > 0, \quad y' < 0$$

at  $(-1, -1)$

$$x' > 0, \quad y' > 0$$

at  $(\frac{1}{4}, 2)$

$$x' > 0, \quad y' = -8 + 4 < 0$$

at  $(\frac{1}{4}, \frac{1}{2})$

$$x' < 0, \quad y' < 0$$

at  $(\frac{1}{4}, -1)$

$$x' < 0, \quad y' > 0$$

at  $(\frac{1}{2}, 2)$

$$x' > 0, \quad y' > 0$$

at  $(1, \frac{1}{2})$

$$x' < 0, \quad y' > 0$$

at  $(1, -1)$

$$x' < 0, \quad y' < 0$$

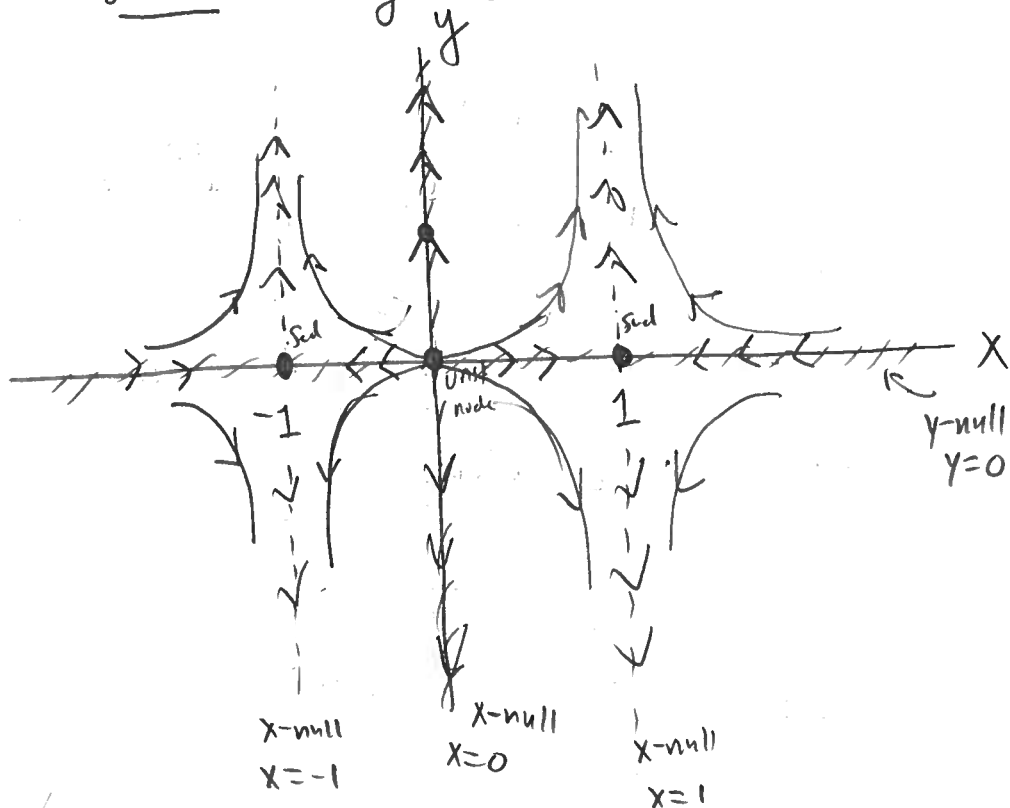
$$\begin{cases} x' = -x + xy \\ y' = -4y + 8xy \end{cases}$$

2

$$\begin{cases} x' = x - x^3 \leftarrow f \\ y' = 2y \leftarrow g \end{cases}$$

x-null:  $0 = x(1-x^2) \rightarrow x=0, x=\pm 1$

y-null:  $0 = 2y \rightarrow y=0$



$$\begin{pmatrix} f_x = 1 - 3x^2 & f_y = 0 \\ g_x = 0 & g_y = 2 \end{pmatrix}$$

at  $(0,0)$

$$J(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$\rightarrow$  evals  $\lambda = 1, 2 \rightarrow$  unstable node

at  $(-1,0)$

$$J(-1,0) = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

$\rightarrow$  evals  $\rightarrow \lambda = -2, 2$   
saddle

at  $(1,0)$

$$J(1,0) = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

evals  $\rightarrow \lambda = -2, 2$   
saddle