Since system

$$\begin{cases} \vec{x}' = ax + by \\ \vec{y}' = cx + dy \end{cases}$$

is 1st order, the geometric approach is useful.

$$\vec{x}' = \begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}}_{\vec{F}(x,y)}$$

vector

NULL CLINE

zero slope

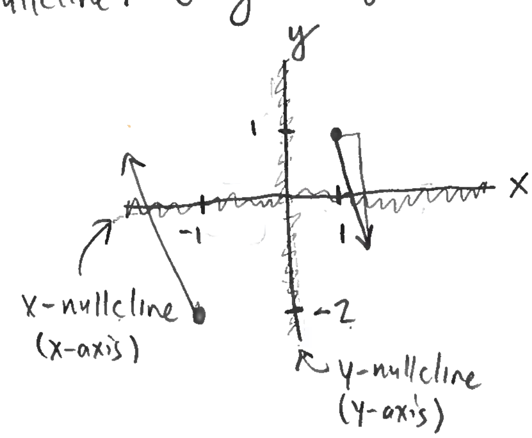
x-nullclines ~ where $x' = 0$, i.e. where $0 = ax + by$

y-nullclines ~ where $y' = 0$, i.e. where $0 = cx + dy$

Ex 4.3 $\begin{cases} x' = y \\ y' = -4x \end{cases}$ ~ $\vec{F} = \begin{pmatrix} y \\ -4x \end{pmatrix}$ $\qquad \vec{F} = (y \;\; -4x)$

x-nullcline: $0 = y$ $\qquad$ y-nullcline: $0 = -4x$

$x = 0$




$$\vec{F}(1,1) = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

x-coordinate

y-coordinate

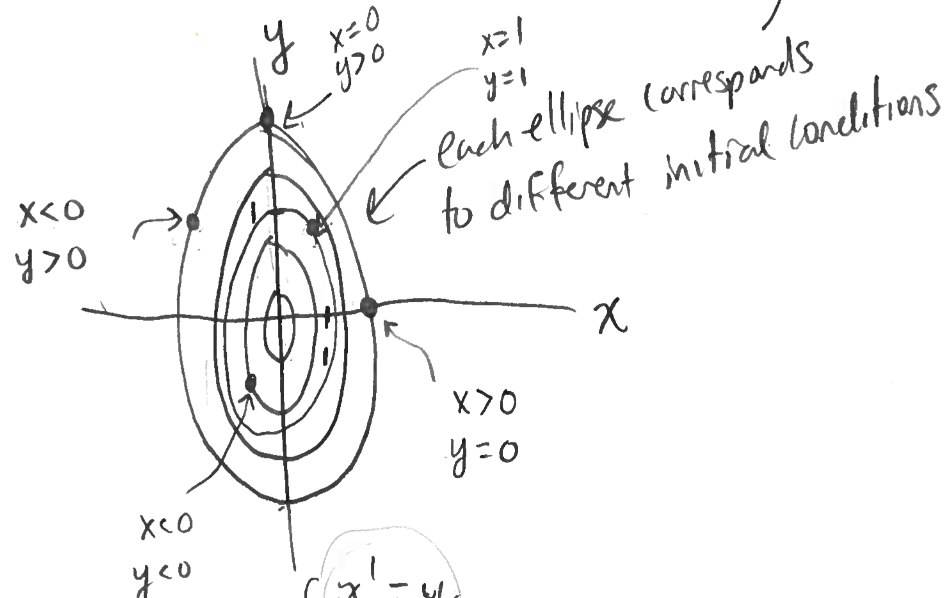
$$\vec{F}(\underset{x}{-1}, \underset{y}{-2}) = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

⟨Slope field calculator on webpage⟩

↳ System tab

↳ type in system we have

(under "Plot numerical soln curves" click RK4)




Back to original eqt!

$$\begin{cases} x' = y \\ y' = -4x \end{cases}$$

Think like calc 1:

① if $y>0$, then $x'>0 \rightsquigarrow x$ is increasing

② if $y<0$, then $x'<0 \rightsquigarrow x$ is decreasing

③ if $x>0$, then $y'<0 \rightsquigarrow y$ is decreasing

④ if $x<0$, then $y'>0 \rightsquigarrow y$ is inc.

Orbits appear to be nested <u>ellipses</u> ← technically can't tell from vec field alone

Turns out a soln to $\begin{cases} x' = y \\ y' = -4x \end{cases}$ is given by

$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{array}{cc} \begin{pmatrix} \cos(2t) \\ -2\sin(2t) \end{pmatrix} & \begin{array}{c} x \\ y \end{array} \end{array}$$

Check:

$$\vec{x}\,' = \begin{pmatrix} \frac{d}{dt}\cos(2t) \\ \frac{d}{dt}\left[-2\sin(2t)\right] \end{pmatrix} = \begin{pmatrix} -2\sin(2t) \\ -4\cos(2t) \end{pmatrix}$$

equal!!

but

$$\begin{pmatrix} y \\ -4x \end{pmatrix} = \begin{pmatrix} -2\sin(2t) \\ -4\cos(2t) \end{pmatrix}$$

* solution oscillates w/ period $\pi$

* can actually obtain orbit equation by "eliminating t": $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos(2t) \\ -2\sin(2t) \end{pmatrix}$

$$x^2 = \cos^2(2t)$$

$$y^2 = 4\sin^2(2t) \longrightarrow \frac{y^2}{4} = \sin^2(2t)$$

SO

$$x^2 + \frac{y^2}{4} = \underbrace{\cos^2(2t) + \sin^2(t)}_{\text{Pyth. identity}} = 1 \Rightarrow \boxed{x^2 + \frac{y^2}{4} = 1}$$

## Equilibria:

The system

$$\begin{cases} x' = ax + by \\ y' = cx + dy \end{cases}$$

<u>always</u> has a solution $\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x(t)$

$y(t)$

"critical point"

b/c $x' = 0, y' = 0$ there

$ax'' + by' + cy = 0$

↑

Similarly, $y = 0$ was a soln

p. 190 #1c

Plot $\begin{cases} x = t^2 - 1 \\ y = 2t \end{cases}$ $t \in \mathbb{R}$