P. 173 #6

$$\begin{cases} x'' + 4x = \delta_2(t) - \delta_5(t) \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

$\downarrow \mathcal{L}$

$$\left(s^2 X(s) - 0 \cdot s - 0\right) + 4X(s) = e^{-2s} - e^{-5s}$$

$$X(s) = \frac{e^{-2s}}{s^2+4} - \frac{e^{-5s}}{s^2+4}$$

$F(s) = \frac{1}{s^2+4}$

$\downarrow$ (k=2)

$f(t) = \frac{1}{2}\sin(2t)$

$\downarrow \mathcal{L}^{-1}$

$$x(t) = \frac{H(t-2)}{2}\sin(2(t-2)) - \frac{H(t-5)}{2}\sin(2(t-5))$$

<plot>

# Ch. 4 Linear Systems

Start w/ damped oscillator

$$mx'' + \gamma x' + kx = 0$$

Goal: find $x$ ← position function of spring

Introduce velocity function $y \overset{def}{=} x'$

$$\Downarrow$$

$$y' = x''$$

$$my' + \gamma y + kx = 0$$

OR

$$y' = -\frac{k}{m}x - \frac{\gamma}{m}y$$

$$\Rightarrow \begin{cases} x' = y \\ y' = -\frac{k}{m}x - \frac{\gamma}{m}y \end{cases}$$

← 1st order system of diff eqts in 2 variables

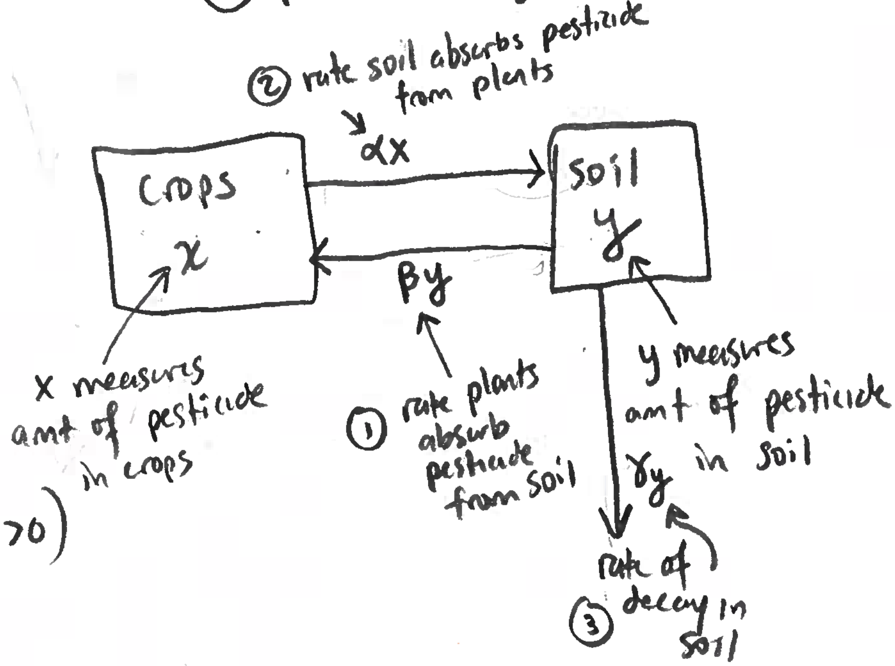
# Ex 4.2 (Compartmental model)

(e.g. coupled chem reactions, physiological models, disease models, etc)

Consider crop + soil surrounding it as seperate "compartments".

Spray pesticide at time $t=0$:

1. it makes its way into plants' roots
2. plants respirate it back into soil
3. pesticide degrades over time in soil

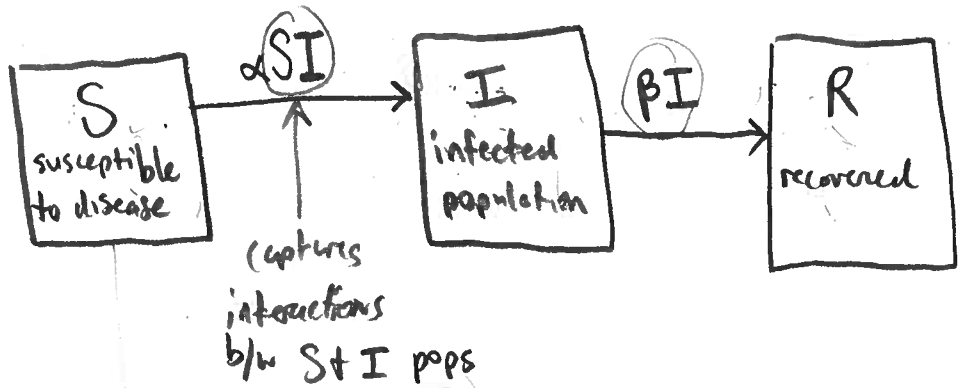



$(\alpha, \beta, \gamma > 0)$

$$\begin{cases} \frac{dx}{dt} = \text{rate of change of pesticide in crops} = -\alpha x + \beta y \\ \frac{dy}{dt} = \text{r.o.c. of pesticide in soil} = \alpha x - \beta y - \gamma y \end{cases}$$

# SIR-model

Disease in a pop. model




$$\begin{cases} \frac{dS}{dt} = -\alpha SI \\ \frac{dI}{dt} = \alpha SI - \beta I \\ \frac{dR}{dt} = \beta I \end{cases}$$

NONLINEAR ($-\alpha SI$)

Linear homogeneous systems of DE's w/ const coeff:

$$\begin{cases} x' = ax + by \\ y' = cx + dy \end{cases} \longleftrightarrow \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$$

(column) vectors

A <u>solution</u> to this is a pair $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ such that when plugged in make all eqtns true.

Generally, $\infty$-many solns until initial conditions are imposed: $x(0) = x_0$, $y(0) = y_0$

Once you have IC's $\rightsquigarrow$ you get unique soln.

Another way to write system

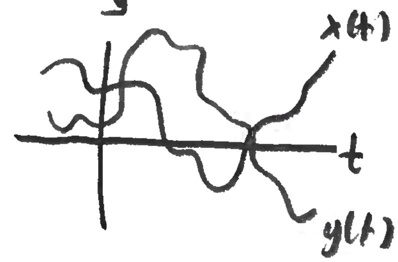
$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{and} \quad \vec{F} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$$

"vector" ($\vec{x}$)

$$\vec{x}' = \vec{F}$$

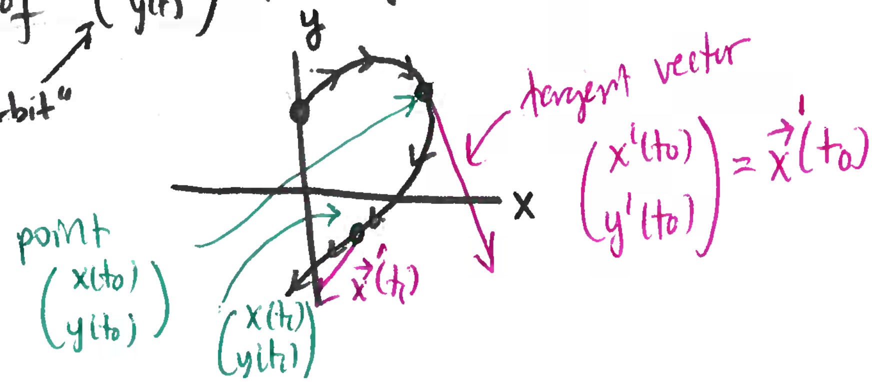
Two ways to graphically represent solns:

① just plot $x(t)$ and $y(t)$ on same plot:



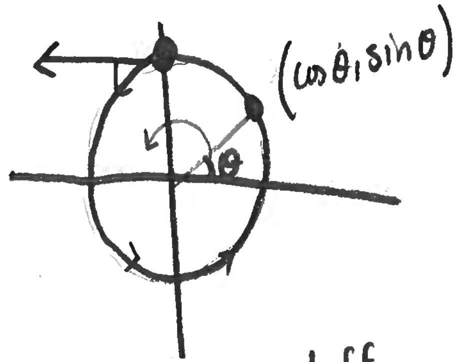

② think of $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ as a parametrized curve

"orbit"



Most popular parametric curve

$$\vec{x} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$




$$\vec{x}\,' = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

$$\vec{x}\,'\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} -\sin(\pi/2) \\ \cos(\pi/2) \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

-1 ← x-stuff

0 ← y-stuff