p.157 #6b

$$\begin{cases} x' + x = \sin(2t) \\ x(0) = 0 \end{cases}$$

$$\downarrow \mathcal{L}$$

$$\underbrace{\left(s X(s) - 0\right)}_{\mathcal{L}\{x'\}} + X(s) = \frac{2}{s^2+2^2}$$

$$X(s) = \frac{2}{(s+1)(s^2+2^2)}$$

partial fract:

$$\frac{2}{(s+1)(s^2+2^2)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2^2}$$

$$0s^2 + 0s + 2 = (A)(s^2+2^2) + \underbrace{(Bs+C)(s+1)}_{Bs^2 + (B+C)s + C}$$

$$= (A+B)s^2 + (B+C)s + (4A+C)$$

$$\begin{cases} A+B = 0 \rightarrow \boxed{A=-B} \\ B+C = 0 \rightarrow \boxed{C=-B} \\ 4A + C = 2 \end{cases}$$

$$\rightarrow -4B - B = 2$$

$$-5B = 2$$

$$\boxed{B = -\frac{2}{5}}$$

$$\boxed{C = 2/5} \quad \boxed{A = 2/5}$$

So,

$$X(s) = \frac{2}{(s+1)(s^2+2)} = \frac{2}{5}\cdot\frac{1}{s-(-1)} - \frac{2}{5}\frac{s}{s^2+2^2} + \frac{1}{5}\cdot\frac{2}{s^2+2^2}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$x(t) = \left(\frac{2}{5}\right)e^{-t} - \frac{2}{5}\cos(2t) + \frac{1}{5}\sin(2t)$$

$\boxed{\#6d}$ $\begin{cases} x'' - 2x' + 2x = 0 \\ x(0) = 0,\ x'(0) = 1 \end{cases}$

$$\boxed{s^2 + bs = \left(s+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2}$$
$$= s^2 + bs + \cancel{\left(\frac{b}{2}\right)^2} - \cancel{\left(\frac{b}{2}\right)^2}$$

$$\Big\downarrow \mathcal{L}$$

$$(s^2X(s) - 0 - 1) - 2(sX(s) - 0) + 2X(s) = 0$$

$$\Big\downarrow$$

$$X(s) = \frac{1}{s^2 - 2s + 2} = \frac{1}{(s-1)^2+1}$$

$F(s) = \dfrac{1}{s^2+1}$

$\downarrow$

$f(t) = \sin(t)$

and

$F(s-1) = \dfrac{1}{(s-1)^2+1}$

does not factor — has complex roots

$\Rightarrow$ complete square $\Rightarrow$

$$\boxed{\begin{aligned} (s^2 - 2s) + 2 &= \left[(s-1)^2 - (-1)^2\right] + 2 \\ \boxed{b = -2} \qquad &= (s-1)^2 + 1 \end{aligned}}$$

Therefore,

$$x(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1}\right\} = \sin(t)e^{t}$$

s^2+1

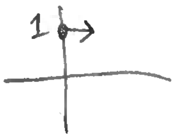
#6f)

$$\begin{cases} x'' - x' = 0 \\ x(0)=1, \ x'(0)=0 \end{cases}$$

$$\Big\downarrow \mathcal{L}$$

$$\left(s^2 X(s) - s - 0\right) - \left(sX(s) - 1\right) = 0$$

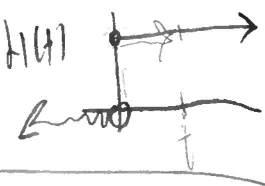
$$X(s) = \frac{s+1}{s^2+s} = \frac{\cancel{s+1}}{s\cancel{(s+1)}} = \frac{1}{s}$$



$$\Big\downarrow \mathcal{L}^{-1}$$

$$x(t) = 1$$

H(t)



#15)

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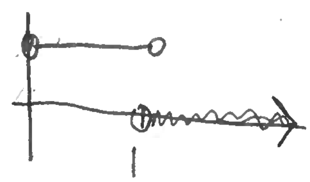
$$\begin{cases} x'' + \pi^2 x = \begin{cases} \pi^2, & 0<t<1 \\ 0, & t>1 \end{cases} \left(= \pi^2 - \pi^2 \underbrace{H(t-1)}_{\text{"turns "on" at } t=1\text{"}} \right) \\ x(0)=1, \ x'(0)=0 \end{cases}$$



$$\Big\downarrow \mathcal{L}$$

$$\left(s^2 X(s) - s - 0\right) + \pi^2 X(s) = \frac{\pi^2}{s} + \frac{\pi^2}{s} e^{-s}$$

$$X(s) = \frac{s}{s^2+\pi^2} + \frac{\pi^2}{s(s^2+\pi^2)} + \frac{\pi^2}{s(s^2+\pi^2)} e^{-s}$$

difficulty: easy, medium, medium+

Partial fract:

$$\frac{\pi^2}{s(s^2+\pi^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+\pi^2}$$

$$\pi^2 = (A)(\cancel{s^2}+\pi^2) + \underbrace{(Bs+C)(s)}_{B\cancel{s^2}+C\cancel{s}}$$

$$0s^2 + 0s + \pi^2 = (A+B)s^2 + (C)s + (A\pi^2)$$

$$\Rightarrow \begin{cases} A+B=0 \longrightarrow \boxed{B=-1} \\ C=0 \\ A\pi^2 = \pi^2 \longrightarrow \boxed{A=1} \end{cases}$$

$$\Rightarrow \frac{\pi^2}{s(s^2+\pi^2)} = \frac{1}{s} + \frac{-s+0}{s^2+\pi^2}$$

Therefore,

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+\pi^2}\right\} + \pi^2\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s}{s^2+\pi^2}\right\} + \pi^2\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s} - \frac{se^{-s}}{s^2+\pi^2}\right\}$$

$$= \cos(\pi t) + \pi^2\left[1 - \cos(\pi t)\right] + \pi^2\left[H(t-1)(1) - H(t-1)\cos(\pi(t-1))\right]$$