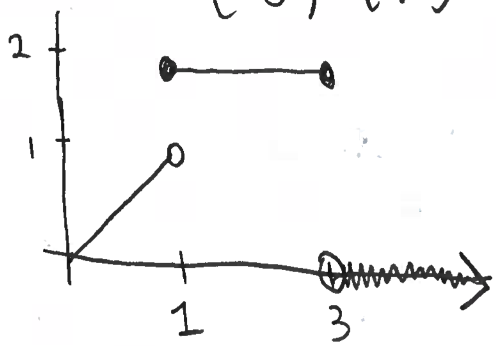
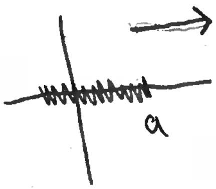
Ex 3.18

$$f(t) = \begin{cases} t, & 0<t<1 \\ 2, & 1 \le t \le 3 \\ 0, & t>3 \end{cases} = t + (\overset{\text{on}}{2} - \overset{\text{off}}{t})H(t-1) - 2H(t-3)$$



H(t-a)



$\mathcal{L}\{t\} = \frac{1}{s^2}$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t\} + \mathcal{L}\{2H(t-1)\} - \mathcal{L}\{t\,H(t-1)\} - \mathcal{L}\{2H(t-3)\}$$

$$= \frac{1}{s^2} + \frac{2}{s}e^{-s} - e^{-s}\mathcal{L}\{\underset{\frac{1}{s^2}}{t} + \underset{\frac{1}{s}}{1}\} - 2\frac{e^{-3s}}{s}$$

$$= \frac{1}{s^2} + \frac{2}{s}e^{-s} - e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right) - 2\frac{e^{-3s}}{s}$$

$$= \frac{1}{s^2} + \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s} - \frac{2}{s}e^{-3s}$$

Ex 3.19 Solve

$\sin(t+\pi) = \sin(t)\cos(\pi) + \cos(t)\sin(\pi) = -\sin(t)$

Sum of angles formula

$$\begin{cases} x'' + 4x = \sin(t) - H(t-\pi)\sin(t) \\ x(0)=0, \quad x'(0)=0 \end{cases}$$

$$\Big\downarrow \mathcal{L}$$

$$\left(s^2 X(s) - s\,x(0) - 0\right) + 4X(s) = \frac{1}{s^2+1} - e^{-\pi s}\mathcal{L}\{\sin(t+\pi)\}$$

$$= \frac{1}{s^2+1} + e^{-\pi s}\cdot\frac{1}{s^2+1}$$

$$s^2 X(s) + 4X(s) = \left(\frac{1}{s^2+1}\right)\left(1+e^{-\pi s}\right)$$

$$X(s) = \frac{1}{(s^2+4)(s^2+1)} + \frac{1}{(s^2+4)(s^2+1)}e^{-\pi s}$$

$$\frac{1}{(s^2+4)(s^2+1)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1}$$

$$0s^3 + 0s^2 + 0s + 1 = (As+B)(s^2+1) + (Cs+D)(s^2+4)$$

$$= \cancel{As^3} + As + \cancel{Bs^2} + \cancel{B} + \cancel{Cs^3} + 4Cs + \cancel{Ds^2} + 4D$$

$$= \underbrace{(A+C)}\,s^3 + (B+D)s^2 + (A+4C)s + (B+4D)$$

$$\begin{cases} A + C = 0 \rightarrow \boxed{A=-C} \\ B + D = 0 \rightarrow \boxed{B=-D} \\ A + 4C = 0 \\ B + 4D = 1 \end{cases}$$

$3C = 0 \rightarrow \boxed{C=0} \rightarrow \boxed{A=0}$

$3D = 1 \rightarrow \boxed{D=\frac{1}{3}} \rightarrow \boxed{B=-\frac{1}{3}}$

So, $\mathcal{L}^{-1}\left\{\frac{1}{s^2+k^2}\right\} = \frac{1}{k}\sin(kt)$

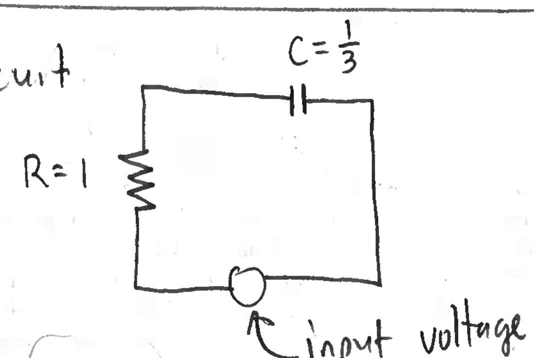
$$X(s) = \left[\frac{-1/3}{s^2+4} + \frac{1/3}{s^2+1}\right] + \overbrace{\left[\frac{-1/3}{s^2+4} + \frac{1/3}{s^2+1}\right]}^{F(s)} e^{-\pi s}$$

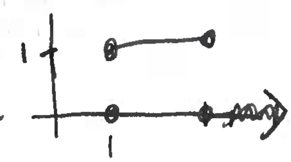
$4 = 2^2$ $\quad$ $1 = 1^2$

$\downarrow \mathcal{L}^{-1}$

$$x(t) = \underbrace{\left[-\frac{1}{6}\sin(2t) + \frac{1}{3}\sin(t)\right]}_{f(t)} + H(t-\pi)\left[-\frac{1}{6}\sin(2(t-\pi)) + \frac{1}{3}\sin(t-\pi)\right]$$

Ex 3.21 $\quad$ RC circuit





$$\Rightarrow \begin{cases} q' + 3q = H(t-1) - H(t-2) \\ q(0) = 0 \end{cases}$$

$\downarrow \mathcal{L}$

$$(sQ(s) - 0) + 3Q(s) = \frac{1}{s}e^{-s} - \frac{1}{s}e^{-2s}$$

$$Q(s) = \frac{1}{s(s+3)}e^{-s} - \frac{1}{s(s+3)}e^{-2s}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+3)}\right\} = \frac{1}{0-(-3)}\left(e^{0t} - e^{-3t}\right) = \frac{1}{3}\left(1 - e^{-3t}\right)$$

$$q(t) = \mathcal{L}^{-1}\{Q\} = H(t-1)\frac{1}{3}\left(1 - e^{-3(t-1)}\right) - H(t-2)\frac{1}{3}\left(1 - e^{-3(t-2)}\right)$$