Ex: Compute

e^{at} with $a=-2$

$$\mathcal{L}\{te^{-2t}\} = F(s-a) = \frac{1}{(s-a)^2}$$

shift prop

$$= \frac{1}{(s+2)^2}$$

earlier we saw

$\mathcal{L}\{t\} = \frac{1}{s^2}$

$f(t)=t$

$F(s)=\frac{1}{s^2}$

$\mathcal{L}\{f\}$

Shift Prop

$\mathcal{L}^{-1}\{F(s-a)\} = f(t)e^{at}$

Ex: Find $\mathcal{L}^{-1}$ of $G(s) = \underbrace{\frac{1}{s-2}}_{F(s)} e^{-3s}$

$a=3$

Switch prop

$\mathcal{L}^{-1}\{e^{-as}F(s)\}$

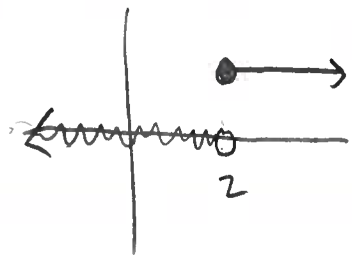
$= H(t-a)f(t-a)$

Recall: $\mathcal{L}^{-1}\left\{\frac{1}{s-b}\right\} = e^{bt}$ $\Rightarrow$ $f(t)=e^{2t}$

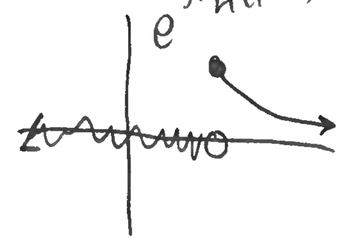
$$\boxed{\mathcal{L}^{-1}\{G\} = H(t-3)e^{2(t-3)}}$$

p.145 #2) Calculate $\mathcal{L}\{e^{-3t}H(t-2)\}$

H(t-2)



$e^{-3t}H(t-2)$



Calculate

$$\int_0^\infty e^{-3t}H(t-2)e^{-st}\,dt$$

$$=\int_2^\infty e^{-3t}e^{-st}\,dt$$

$$=\int_2^\infty e^{-(3+s)t}\,dt$$

$$=\lim_{b\to\infty}\int_2^b e^{-(3+s)t}\,dt$$

$$=\lim_{b\to\infty}\frac{1}{-(3+s)}e^{-(3+s)t}\Big|_{t=2}^{t=b} = \lim_{b\to\infty}\frac{e^{-(3+s)b}}{-(3+s)}+\frac{1}{3+s}e^{-(3+s)(2)}$$

$\to 0$ requires $-(3+s)<0$

$3+s>0$

$s>-3$

$$\overset{(s>-3)}{=}\frac{1}{3+s}e^{-2(3+s)}$$

#3 Recall

$$\sin(kt) = \frac{e^{ikt} - e^{-ikt}}{2i} \qquad \cos(kt) = \frac{e^{ikt} + e^{-ikt}}{2}$$

Recall

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad \mathcal{L}\{\overset{\text{const}}{\alpha f + \beta g}\} = \alpha\mathcal{L}\{f\} + \beta\mathcal{L}\{g\}$$

So,

$$\mathcal{L}\{\sin(kt)\} = \mathcal{L}\left\{\frac{e^{ikt}}{2i} - \frac{e^{-ikt}}{2i}\right\}$$

$i^2 = -1$

$$= \frac{1}{2i}\mathcal{L}\{e^{ikt}\} - \frac{1}{2i}\mathcal{L}\{e^{-ikt}\}$$

$$= \frac{1}{2i}\frac{1}{s-ik} - \frac{1}{2i}\frac{1}{s+ik}$$

$$= \frac{1}{2i}\left[\frac{(\cancel{s}+ik)-(\cancel{s}-ik)}{(s-ik)(s+ik)}\right] = \frac{1}{\cancel{2i}}\left[\frac{\cancel{2i}k}{s^2-i^2k^2}\right]$$

$$= \frac{k}{s^2+k^2}$$

$$\mathcal{L}\{\cos(kt)\} = \mathcal{L}\left\{\frac{e^{ikt}+e^{-ikt}}{2}\right\}$$

$$= \frac{1}{2}\cdot\frac{1}{s-ik} + \frac{1}{2}\frac{1}{s+ik}$$

$$= \frac{1}{2}\left[\frac{(s+\cancel{ik})+(s-\cancel{ik})}{s^2+k^2}\right] = \frac{s}{s^2+k^2}$$

#8a) $\mathcal{L}\{6+5e^{-2t}+te^{3t}\}$ $\quad$ "f(t)" $\rightarrow F(s)=\frac{1}{s^2}$

a=3 $\quad F(s-a)$

$=6\mathcal{L}\{1\}+5\mathcal{L}\{e^{-2t}\}+\mathcal{L}\{te^{3t}\}$ $\quad =F(s-3)$

$=\frac{6}{s}+\frac{5}{s+a}+\underbrace{\frac{1}{(s-3)^2}}_{\text{shift prop}}$

#8b) $\mathcal{L}\{tH(t-3)\}=\underbrace{\frac{1}{s^2}e^{-3s}}_{\text{switching prop}}$

$f(t)=t$ $\quad$ a=3 for switching prop

$F(s)=\frac{1}{s^2}$

#9a) $\mathcal{L}^{-1}\left\{\frac{7}{s+2}\right\}=7\mathcal{L}^{-1}\left\{\frac{1}{s-(-2)t}\right\}=7e^{-2t}$ $\quad$ a=-2

$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\}=e^{at}$

$\mathcal{L}\{e^{at}\}=\frac{1}{s-a}$

$\mathcal{L}\{\sin(kt)\}=\frac{k}{s^2+k^2}$

$\frac{1}{k}\mathcal{L}\{\sin(kt)\}=\frac{1}{s^2+k^2}$

$\frac{1}{k}\sin(kt)=\mathcal{L}^{-1}\left\{\frac{1}{s^2+k^2}\right\}$

#9f) $\mathcal{L}^{-1}\left\{\frac{7s+1}{s^2+4}\right\}=7\underbrace{\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}}_{\cos}+\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}}_{\sin}$

$=7\mathcal{L}^{-1}\left\{\frac{s}{s^2+2^2}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{s^2+2^2}\right\}$

$=7\cos(2t)+\frac{1}{2}\sin(2t)$