Ex 2.23

$$x'' + 3x' + 3x = 6e^{-2t} + 4$$

homogeneous: $\lambda^2 + 3\lambda + 3 = 0$

$$\lambda = \frac{-3 \pm \sqrt{9-4(1)(3)}}{2}$$

$$= -\frac{3}{2} \pm \frac{1}{2}\sqrt{-3} = \underset{\alpha}{-\frac{3}{2}} \pm \underset{\beta}{\frac{\sqrt{3}}{2}}\, i$$

$$\Rightarrow \quad X_h(t) = C_1 e^{-\frac{3}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{-3/2t} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$x_{p_1}(t) = Ae^{-2t} + B$$

$$x(t) = x_h(t) + 6e^{-2t} + \frac{4}{3}$$

$$x'_{p_1} = -2Ae^{-2t}$$

$$x''_{p_1} = 4Ae^{-2t}$$

$$4Ae^{-2t} + -6Ae^{-2t} + 3Ae^{-2t} + 3B = 6e^{-2t} + 4$$

$$[4A - 6A + 3A]e^{-2t} + 3B = 6e^{-2t} + 4$$

$$Ae^{-2t} + 3B = 6e^{-2t} + 4 \Rightarrow A = 6$$

$$B = 4/3$$

Ex 3.24:

RCL ~ $R=2, C=L=1, f(t)=2\sin(3t)$

$$\Rightarrow \begin{cases} Q''+2Q'+Q=2\sin(3t) \\ Q(0)=4,\ Q'(0)=0 \end{cases}$$

Step 1: Solve $\begin{cases} Q''+2Q'+Q=0 \\ Q(0)=4,\ Q'(0)=0 \end{cases}$

$$\Rightarrow \lambda^2+2\lambda+1=0 \rightarrow (\lambda+1)^2=0$$

$$\lambda=-1$$

$$\Rightarrow x_h(t)=c_1e^{-t}+c_2te^{-t}$$

IC's $\rightarrow$ $c_1=4.123 \quad c_2=4.597$

Step 2: $Q_p(t)=A\sin(3t)+B\cos(3t)$

$$\rightarrow Q_p'(t)=3A\cos(3t)-3B\sin(3t)$$

$$Q_p''(t)=-9A\sin(3t)-9B\cos(3t)$$

$$[-9A\,s(3t)-9B\,c(3t)]+[6A\cos(3t)-6B\,s(3t)]+[A\,s(3t)+B\,c(3t)]=2s(3t)$$

$$\boxed{[-8A-6B]\sin(3t)}+\boxed{[-8B+6A]\cos(3t)}=\boxed{2\sin(3t)}+\boxed{0\cos(3t)}$$

$$\Rightarrow \begin{cases} -8A-6B=2 \\ -8B+6A=0 \end{cases} \Rightarrow A=-4/25 \quad B=-3/25$$

EX 2.25:

$$Q'' + 9Q = \sin(3t)$$

Step 1: $\lambda^2 + 9 = 0$

$\lambda = \pm 3i$

$$x_h(t) = c_1\cos(3t) + c_2\sin(3t)$$

Step 2: $x_p(t) = At\sin(3t) + Bt\cos(3t)$

$x_p' = A\sin(3t) + A3t\cos(3t)$
$+ B\cos(3t) - B3t\sin(3t)$

$x_p'' = A3\cos(3t) + A3C(3t) - A9ts(3t) - 3Bs(3t)$
$- B3\sin(3t) - B9t\cos(3t)$

$= [6A]\cos(3t) - 6Bs(3t) - 9Ats(3t) - 9Bt\cos(3t)$

$= (6A - 9\cancel{Bt})\cos(3t) + (-6B - 9\cancel{At})\sin(3t)$

So, $x_p'' + 9x_p = 6A\cos(3t) - 6B\sin(3t) = \sin(3t)$

$\boxed{A=0} \leftarrow 6A = 0$

$-6B = 1 \rightarrow \boxed{B = -\frac{1}{6}}$