b) Solve

$x' + p(t)x = q(t)$

$$S' = I\left(1 - \frac{S}{P}\right) - \frac{E}{P}S$$

$$\int e^{2t}\,dt = \frac{1}{2}e^{2t}$$

$$S' + \left(\frac{I+E}{P}\right)S = I$$

$$\mu(t) = e^{\int \frac{I+E}{P}dt} = e^{t\left(\frac{I+E}{P}\right)}$$

$$\left(Se^{t\left(\frac{I+E}{P}\right)}\right)' = Ie^{t\left(\frac{I+E}{P}\right)}$$

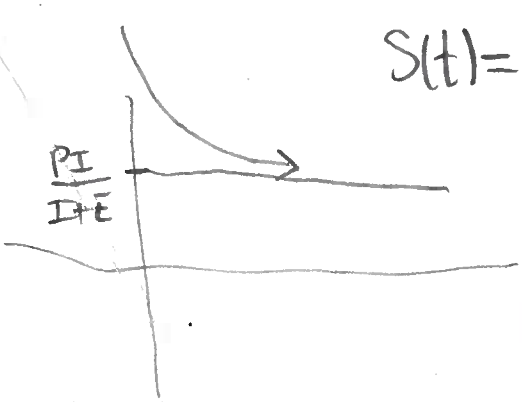
$$Se^{t\left(\frac{I+E}{P}\right)} = \frac{I}{\frac{I+E}{P}}\cancel{e^{t\left(\frac{I+E}{P}\right)}} + C$$

$$S(t) = \frac{PI}{I+E} + Ce^{-t\left(\frac{I+E}{P}\right)}$$

$$\underbrace{S_0}_{\text{given}} = S(0) = \frac{PI}{I+E} + C$$

$$\boxed{C = S_0 - \frac{PI}{I+E}}$$

$$S(t) = \frac{PI}{I+E} + \left(S_0 - \frac{PI}{I+E}\right)e^{-t\left(\frac{I+E}{P}\right)}$$



$$\chi_L = I\left(1-\frac{S}{P}\right)$$

$$\mu_L = \frac{E_L}{P}S$$

$$\chi_B = I\left(1-\frac{S}{P}\right)$$

$$\mu_B = \frac{E_B}{P}S$$

Assume: $\boxed{E_B < E_L}$

equilibria on both islands:

Big	Little
$\frac{IP}{I+E_B}$	$\frac{IP}{I+E_L}$

Add I

$$I+E_B < I+E_L$$

Reciprocal

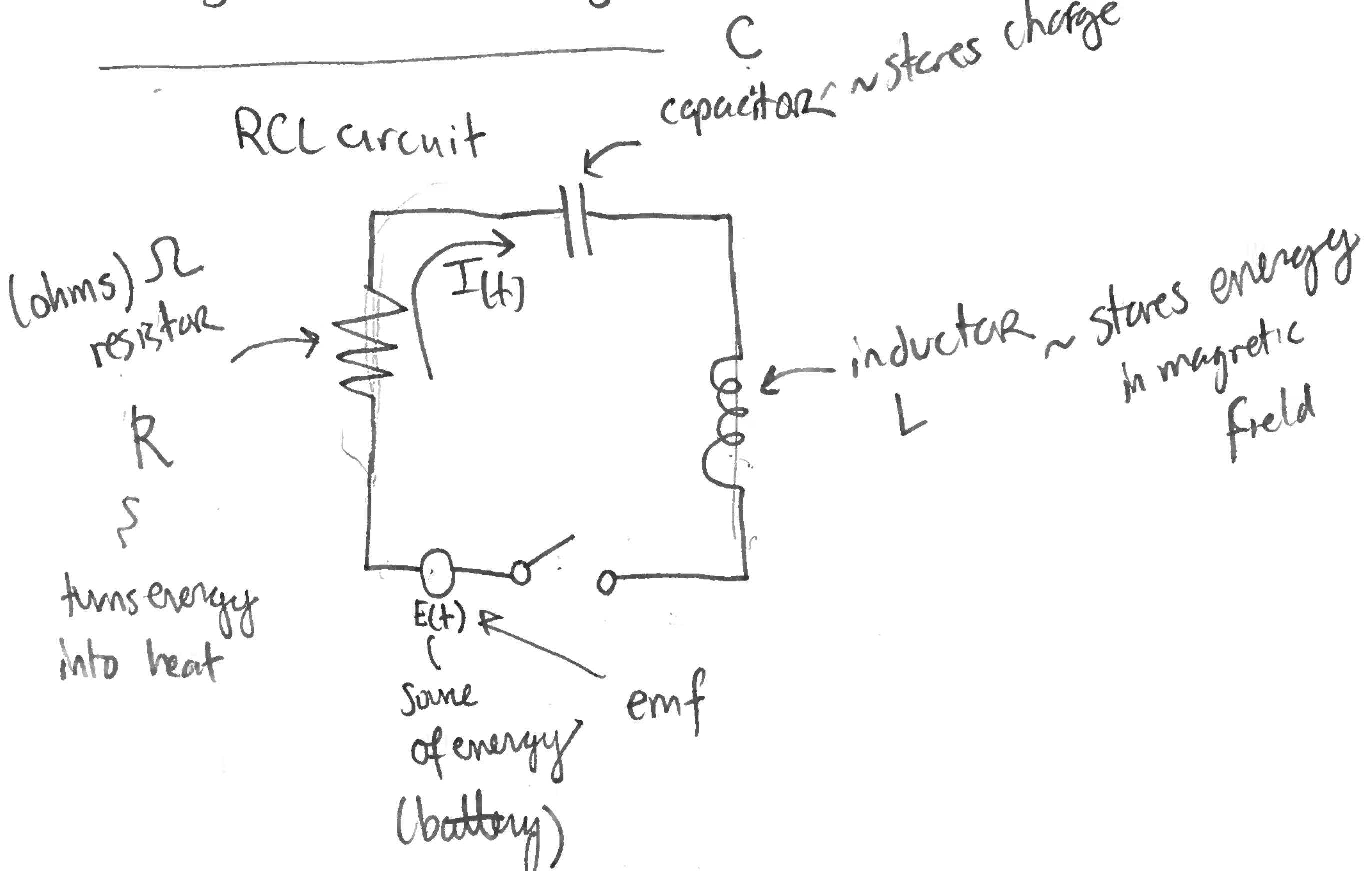
$$\frac{1}{I+E_B} > \frac{1}{I+E_L}$$

Multiply IP

$$\boxed{\frac{IP}{I+E_B} > \frac{IP}{I+E_L}}$$

§1.4.3 Electricity

RCL circuit



Kirchhoff's Law: voltage across circuit in a loop equals applied emf

$$\boxed{V_R + V_L + V_C = E(t)}$$

$\boxed{Q'(t) = I(t)}$ $\begin{cases} Q(t) \sim \text{charge in capacitor} \\ I(t) \sim \text{amperage (current)} \end{cases}$

$\boxed{Q''(t) = I'(t)}$

Ohm's Law: voltage drop across resistor is proportional to current $\quad V_R = RI$

Faraday's Law: voltage drop across inductor is prop. to ~~how~~ fast current changes $\quad V_L = LI'$

<third one>: voltage drop across capacitor is prop. to charge on capacitor $\quad V_C = \frac{1}{C} Q$

Plug V_R, V_L, and V_C into Kirchhoff's Law:

$$RI + LI' + \frac{1}{C}Q = E(t)$$

$$\Big\downarrow \; I = Q', \; I' = Q''$$

Second order diffeq →

$$\boxed{RQ' + LQ'' + \frac{1}{C}Q = E(t)}$$

$$\Big\downarrow \; \frac{d}{dt}$$

$$RQ'' + LQ''' + \frac{1}{C}Q' = E'(t)$$

$$\Big\downarrow \; Q' = I, \; Q'' = I', \; Q''' = I''$$

$$\boxed{RI' + LI'' + \frac{1}{C}I = E'(t)}$$ ← RCL circuit equation

No inductor ~ $L=0$

$$\boxed{RI' + \frac{1}{C}I = E'(t)}$$

Ex 1.29: $R=1$, $C=\frac{1}{2}$, $Q(0)=0$ and emf $E(t) = \begin{cases} 1, & 0 \le t < 2 \\ 0, & t > 2 \end{cases}$

$Q_1' + 2Q = 1$

$\downarrow$

→ $\boxed{Q_1(t) = \frac{1}{2}(1 - e^{-2t})}$

$Q_2' + 2Q = 0$

$\Rightarrow Q_2(t) = Ae^{-2t}, \; t > 2$

need init cond at time $t=2$!

$Q_2(2) = Q_1(2)$

So we have

$$\underbrace{\tfrac{1}{2}(1-e^{-4})}_{\text{from } Q_1} = Q_2(2) = \underbrace{A e^{-4}}_{\text{from } Q_2}$$

$$\downarrow$$

$$A = \tfrac{1}{2}(e^{4}-1)$$

Therefore

$$Q(t) = \begin{cases} \frac{1}{2}(1-e^{-2t}), & 0 \le t \le 2 \\ \frac{1}{2}(e^{4}-1)e^{-2t}, & t > 2 \end{cases}$$

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