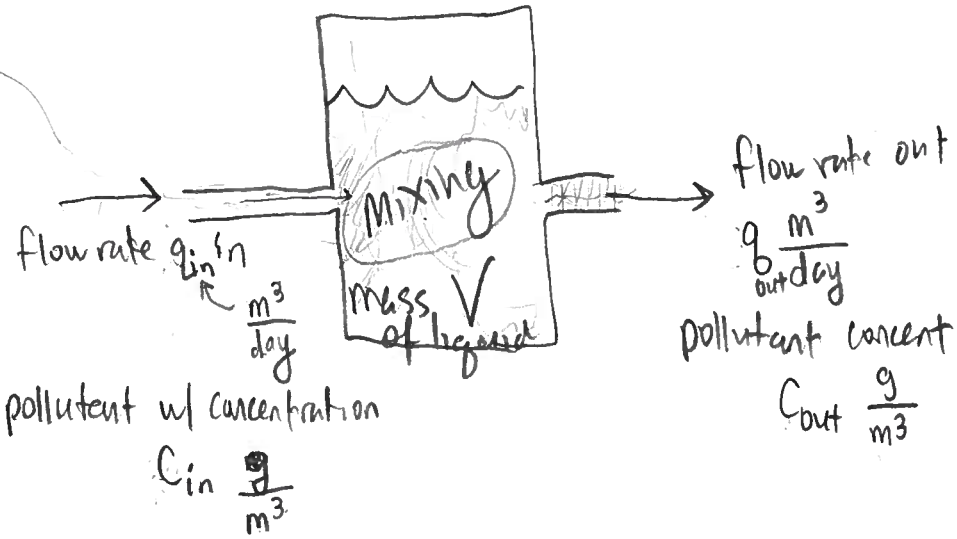
Chemical Reactors



Q: What is concentration of pollutant in vessel at time t? ← call this $C(t)$

Conservation of mass: ★ rate of change of mass of chemical present = mass of chemical flowing in − mass of chemical flowing out

hr
$x(t)$ miles
$x'(t)$ $\frac{miles}{hour}$ vel
$x''(t)$ $\frac{mi}{hr^2}$ accel

★ total mass of chemical in pond = $V \cdot C(t)$ ← units g
(V: m^3, $C(t)$: $\frac{g}{m^3}$)
↑ volume of total liquid

$(\star) \rightarrow (VC)' = q_{in}C_{in} - q_{out}C$ ← $C_{out} = C$ here

units $\frac{g}{day}$

Simplify & think $q_{in} = q_{out} =: q$
$\Rightarrow V$ constant

$$VC' = qC_{in} - qC, \quad C(0) = C_0$$

0 1 2 0.5

If flow rate in + out are not equal

$$q_{in} \neq q_{out}$$

$$(*) \rightarrow (VC)' = q_{in}C_{in} - q_{out}C_{out}, \quad V(t) = V_0 + (q_{in} - q_{out})t$$

(V_0: initial volume)

$(VC)'$: $V'C + C'V$

$$C'(V_0 + (q_{in} - q_{out})t) = q_{in}C_{in} - q_{out}C - (q_{in} - q_{out})C$$

Different assumptions:

If pollutant degrades (proportional) to amount present $\sim$ decay rate $kC \ \frac{g}{m^3 \cdot day}$

(k: units $\frac{1}{day}$)

$q_{in} = q_{out} = q$

$$\boxed{VC' = qC_{in} - qC - kCV}$$

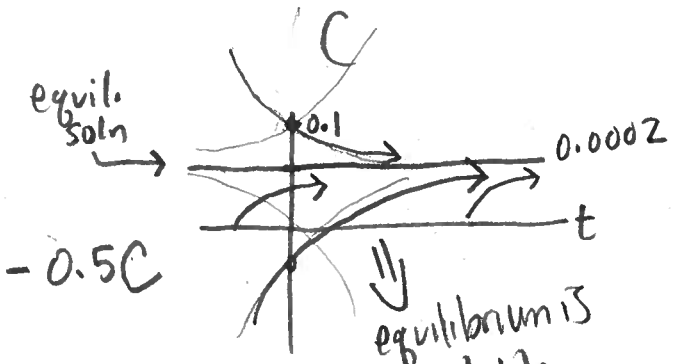
Ex. 2 p. 34: $q_{in} = q_{out} = 0.5 \ \frac{m^3}{min}$

$V = 100 m^3$ (constant)

$C_{in} = 0.0002 \ \frac{kg}{m^3}$

IVP:

$$\begin{cases} 100C' = (0.5)(0.0002) - 0.5C \\ C(0) = 0 \end{cases}$$



equilibrium is stable — solns go toward it!!

equilibrium means no change $\rightsquigarrow C' = 0$

Find equilibrium: set $C' = 0$ in eqt + solve for C

$$0 = 0.5(0.0002) - 0.5C \rightarrow \boxed{C = 0.0002}$$

if as test pt we consider a concentration C of 0.1:

$$100C' = (0.5)(0.0002) - 0.5(0.1) < 0$$

Similarly if $C < 0.0002$, then $C' > 0$

Ex 3 p. 34:

Volume $V(t) = 100 + (0.5 - 0.6)t = 100 - 0.1t$

How long to empty pond?

$$0 = V(t_e) = 100 - 0.1t_e$$

$$t_e = \frac{100}{0.1} = 1000 \text{ minutes}$$

$$\left((100 - 0.1t)C\right)' = 0.5(0.0002) - 0.6C$$

$$-0.1C + (100 - 0.1t)C' = \uparrow$$

$$\begin{cases} C' = \dfrac{(0.5)(0.0002) - 0.5C}{100 - 0.1t} \\ C(0) = 0 \end{cases}$$