p.27 #11

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Solve $x' = \frac{(4+2t)x}{\ln(x)}$ $\quad g(t) = 4+2t$

$f(x) = \frac{x(t)}{\ln(x(t))}$

$$\frac{\ln(x)}{x}\frac{dx}{dt} = 4+2t$$

$u = \ln(x)$

$du = \frac{1}{x}dx$

$$\int \frac{\ln(x)}{x}dx = \int 4+2t\,dt$$

$$\int u\,du = 4t+t^2+C$$

$$\frac{u^2}{2} = 4t+t^2+C$$

$$\frac{(\ln(x))^2}{2} = 4t+t^2+C$$

When $x(0)=e$ → $\frac{(\ln(e))^2}{2} = 0+0+C$

t-value x-value

$$\boxed{\frac{1}{2} = C}$$

must be!

$e = x(0) = e^{\pm\sqrt{0+0+1}}$

↑given shown

particular soln: $\frac{(\ln(x))^2}{2} = 4t+t^2+\frac{1}{2}$

$(\ln(x))^2 = 8t+2t^2+1$

$\ln(x) = \pm\sqrt{8t+2t^2+1}$

which?

$x = e^{\pm\sqrt{8t+2t^2+1}}$

take + soln

(#4 c) $(2u+1)u' - (t+1) = 0$

unknown fnct is u

$$(2u+1)\frac{du}{dt} = t+1$$

$$\int (2u+1)\,du = \int (t+1)\,dt$$

implicit soln $\longrightarrow$ $u^2 + u = \frac{t^2}{2} + t + C$

$$\left(u+\frac{1}{2}\right)^2 - \frac{1}{4} = \frac{t^2}{2} + t + C$$

$$\left(u+\frac{1}{2}\right)^2 = \frac{t^2}{2} + t + \tilde{C} \ ; \ \tilde{C} = C + \frac{1}{4}$$

explicit soln $\longrightarrow$ $u = \pm\sqrt{\frac{t^2}{2} + t + \tilde{C}} - \frac{1}{2}$

Completing the square

$$x^2 + bx = \underbrace{\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2}_{x^2 + bx + \cancel{\left(\frac{b}{2}\right)^2} - \cancel{\left(\frac{b}{2}\right)^2}}$$

✓

§1.3.2 Heat Transfer

Assume object at a constant uniform temp + place it in environment at temp T_e

Goal: model temperature as a funct of time

Newton's law of cooling (or heating): rate of change of temp. is proportional to difference between temp of object + temp of environment

T is temp of object at time t

$$\frac{dT}{dt} = (-h)(T - T_e)$$

$h > 0$ is called heat loss coeff ~ measures how object releases or absorbs heat

Notice:

* $T = T_e$ (constant) is a constant soln ← equilibrium soln

  b/c

  $LHS = T' = (T_e)' = 0$

  $RHS = -h(T_e - T_e) = 0$ match!

* if $T > T_e$, then $T - T_e > 0$, so $T' < 0$ (i.e. temp goes down)
* if $T < T_e$, then $T - T_e < 0$, so $T' > 0$ (i.e. temp goes up)

Solve it: $T' = -h(T - T_e)$

constant

$$\frac{1}{T-T_e}\frac{dT}{dt} = -h$$

$$\int \frac{1}{T-T_e}\,dT = -\int h\,dt$$

$$\ln(|T-T_e|) = -ht + C$$

$$|T-T_e| = e^{-ht+C}$$

$$T - T_e = \tilde{C}e^{-ht} \quad ; \quad \tilde{C} = \pm e^{C} \text{ (dep on sign of } T-T_e\text{)}$$

$$\boxed{T = \tilde{C}e^{-ht} + T_e}$$

if $\tilde{C} > 0 \rightsquigarrow$ mean that object starts at higher temp than environment

if $\tilde{C} < 0 \rightsquigarrow$ ——— lower ———

Impose initial condition $T(0) = T_0$

$$\underset{\text{given}}{T_0} = T(0) = \underbrace{\tilde{C}e^{0} + T_e}_{\text{calculated}} \longrightarrow \tilde{C} = T_0 - T_e$$

$$\Rightarrow T(t) = (T_0 - T_e)e^{-ht} + T_e$$

$e^{-ht} \to 0$ as $t \to \infty$