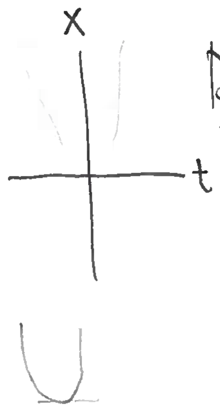
Special places: set of points in xt-plane where the slope is zero is called nullclines



Def: nullclines $= \{(t,x) : x' = f(t,x) = 0\}$

isoclines $= \{(t,x) : x' = f(t,x) = k\}$

constant k

Ex 1.6: $x' = -tx + x^2$

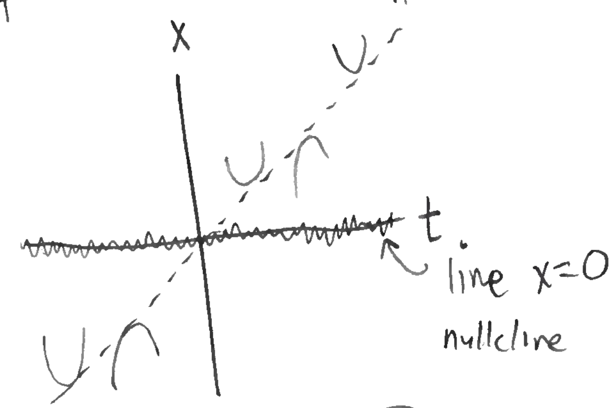
f

nullclines

$-tx + x^2 = 0$

$x(-t+x) = 0$

$x = 0$ OR $x = t$



notice: $x=0$ is a soln

because

LHS $= x' = 0$ ✓

RHS $= -t(0) + 0^2 = 0$ ✓

$x = t$ not a soln

LHS $= x' = 1$

RHS $= -tx + x^2$

$= -t^2 + t^2$

$= 0$

do not match!

⇓

not a soln

Ex 1.7: $x' = x^3 - xt$

nullclines ($x'=0$)

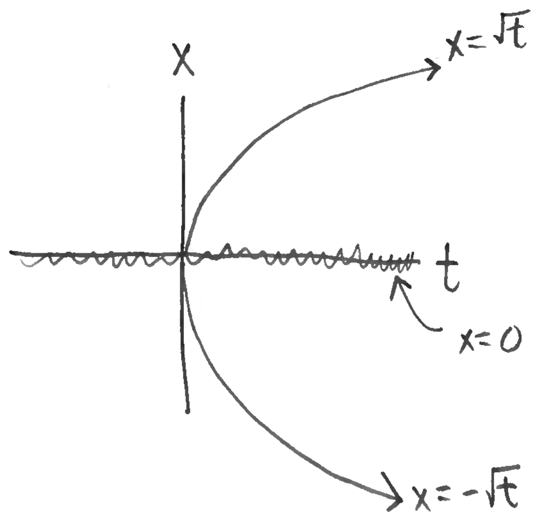
$$0 = x^3 - xt$$

$$0 = x(x^2 - t)$$

$x = 0$ $\qquad$ $x^2 - t = 0$

$$x^2 = t$$

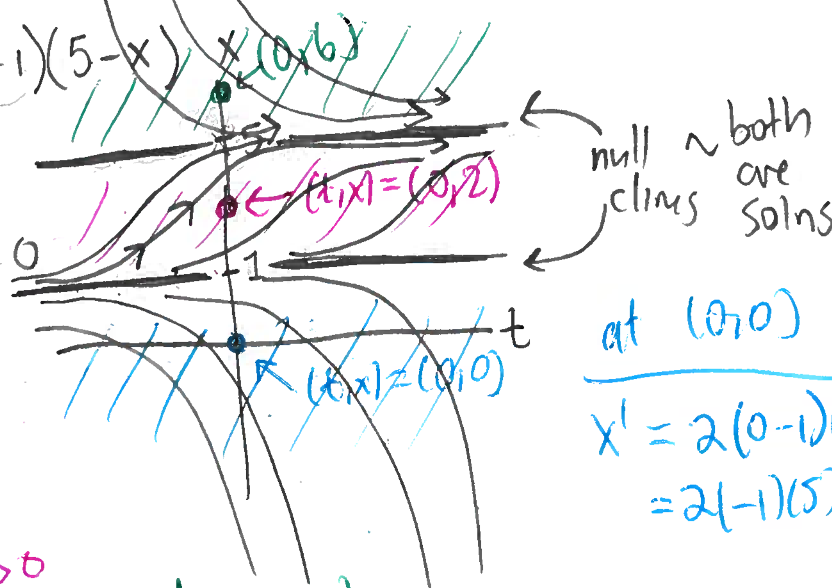
$$x = \pm\sqrt{t}$$



Ex 1.8 ("autonomous")

no "t" appearing on RHS

$$x' = 2(x-1)(5-x)$$



nullclines

$$2(x-1)(5-x) = 0$$

$$x = 1, 5$$

at test pt $(0,2)$

$$x' = 2(2-1)(5-2)$$
$$= 2(1)(3) = 6 > 0$$

at $(0,0)$

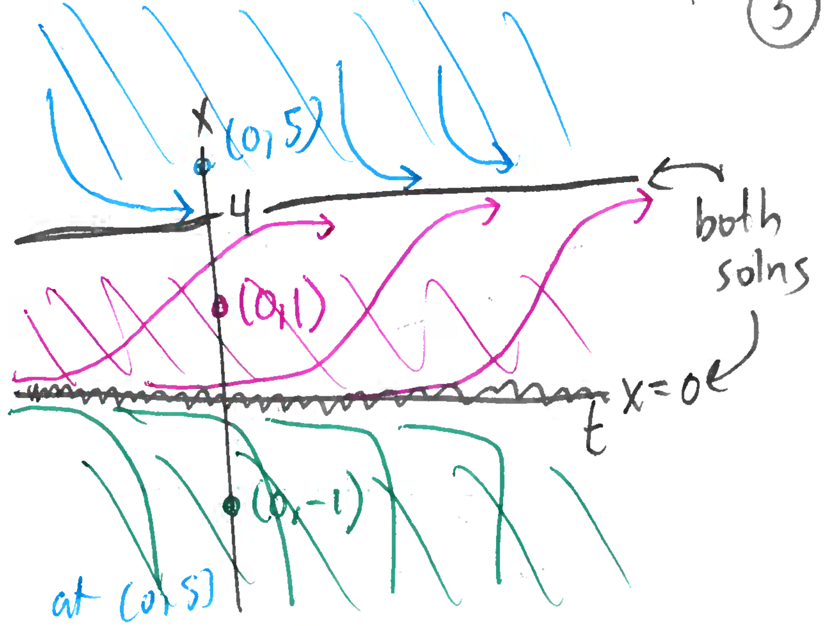
$$x' = 2(0-1)(5-0)$$
$$= 2(-1)(5) = -10 < 0$$

at $(0,6)$

$$x' = 2(5)(-1) = -10 < 0$$

Exercise #1

$$x' = x\left(1-\frac{x}{4}\right)$$



nullclines

$x=0$, $\quad 1-\frac{x}{4}=0$

$x=4$

at $\overset{t}{(}0,\overset{x}{1})$

$$x' = 1\left(1-\frac{1}{4}\right) > 0$$

at $(0,5)$

$$x' = 5\left(1-\frac{5}{4}\right) < 0$$

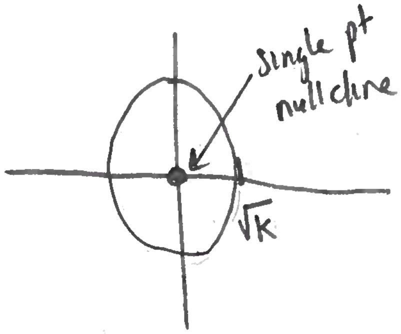
at $(0,-1)$

$$x' = (-1)\left(1+\frac{1}{4}\right) < 0$$

#2) $x' = x^2 + t^2$

understand isoclines: $x' = k\,(>0)$ constant

$x^2 + t^2 = k$ circle of radius $\sqrt{k}$ centered at $(0,0)$



$\cancel{x = \pm\sqrt{k-t^2}}$

if $k=0$ (nullcline)

$0 = x^2 + t^2$

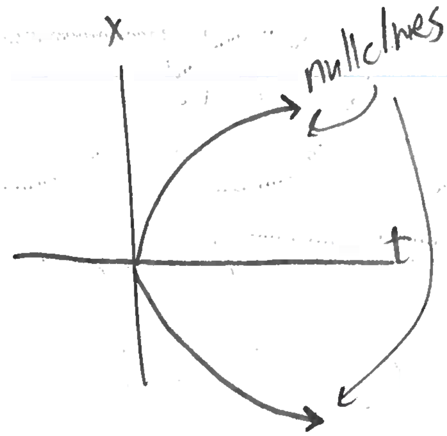
→ only soln $(t,x) = (0,0)$

#4) nullclines of

$$x' = t - x^2$$

$$x' = 0 \rightarrow \quad 0 = t - x^2$$

$$x = \pm\sqrt{t}$$



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§1.2 Antiderivatives

"ordinary" ~ only one dep variable ~ t

Easiest ODEs to solve

$$x'(t) = g(t) \qquad (1.7)$$

Ex: $x' = t + 1$

$\int x' \, dt = x$

$$x = \int t + 1 \, dt = \frac{t^2}{2} + t + C$$

General soln to (1.7) is

$$x(t) = \int g(t)\,dt + C$$

Solve IVP

Ex: $\begin{cases} x' = t^2 - 1 \\ x(1) = 2 \end{cases}$ $\longrightarrow x = \int t^2 - 1 \, dt = \frac{t^3}{3} - t + C$

$$\underset{\text{given}}{2} = x(1) = \underbrace{\tfrac{1}{3} - 1 + C}_{\text{calc}} \rightarrow 2 = -\tfrac{2}{3} + C$$

$$\rightarrow C = 2 + \tfrac{2}{3} = \tfrac{8}{3}$$

$$\Rightarrow \boxed{x(t) = \frac{t^3}{3} - t + \frac{8}{3}}$$