

#C2) Show the following sentences
are jointly satisfiable:

$$\checkmark L(e, e), \checkmark L(e, g), \checkmark \neg L(g, e), \checkmark \neg L(g, g)$$

Solu: We have to construct an interpretation
which makes them all true.

interp: domain: 1, 2

$$L(x, y): \{1, 2\} \rightarrow \{1, 2\}$$

$$e: 1$$

$$g: 2$$

This interpretation works!

#C3) Same instructions:

$$\checkmark \neg(M(a) \wedge \exists x A(x)), \checkmark M(a) \vee F(a), \checkmark \forall x (F(x) \rightarrow A(x))$$

interpretation: domain: 1, 2

$$M(x): M(1) \text{ false}, M(2) \text{ true}$$

$$A(x): A(1) \text{ true}, A(2) \text{ true}$$

$$F(x): F(1) \text{ true}, F(2) \text{ false}$$

$$a: 1$$

← irrelevant which
truth values these
two false

2

#C8) Same instructions

$$\forall x (P(x) \vee Q(x)), \quad \exists x \neg (Q(x) \wedge P(x))$$

Interpretation: domain: 1, 2

$P(x)$: $P(1), P(2)$ both true

$Q(x)$: $Q(1), Q(2)$ both false

#C11) Same

$$\neg R(a, a), \quad \forall x (x = a \vee R(x, a))$$

Interpretation: domain: 1, 2

$R(x, y)$: $1 \leftarrow 2$

a : 1

Prove arguments from A in p. 211

③

A11 $\forall x(G(x) \rightarrow F(x)), \forall x(H(x) \rightarrow G(x)), \therefore \forall x(H(x) \rightarrow F(x))$
Premises

| | | |
|-------|------------------------------------|----------------------|
| 1 | $\forall x(G(x) \rightarrow F(x))$ | |
| 2 | $\forall x(H(x) \rightarrow G(x))$ | |
| <hr/> | | |
| 3 | $G(a) \rightarrow F(a)$ | $\forall E 1$ |
| 4 | $H(a) \rightarrow G(a)$ | $\forall E 2$ |
| 5 | $H(a)$ | |
| <hr/> | | |
| 6 | $G(a)$ | $\rightarrow E 4, 5$ |
| 7 | $F(a)$ | $\rightarrow E 3, 6$ |
| 8 | $H(a) \rightarrow F(a)$ | $\rightarrow I 5-7$ |
| 9 | $\forall x(H(x) \rightarrow F(x))$ | $\forall I 8$ |

A4) $\forall x(G(x) \rightarrow F(x)), \exists x(H(x) \wedge G(x)) \therefore \exists x(H(x) \wedge F(x))$

| | | | |
|-------|--|------------------------------------|----------------------|
| 1 | | $\forall x(G(x) \rightarrow F(x))$ | |
| 2 | | $\exists x(H(x) \wedge G(x))$ | |
| <hr/> | | | |
| 3 | | $H(a) \wedge G(a)$ | |
| <hr/> | | | |
| 4 | | $G(a)$ | $\wedge E 3$ |
| 5 | | $G(a) \rightarrow F(a)$ | $\forall E 1$ |
| 6 | | $F(a)$ | $\rightarrow E 5, 4$ |
| 7 | | $H(a)$ | $\wedge E 3$ |
| 8 | | $H(a) \wedge F(a)$ | $\wedge I 7, 6$ |
| 9 | | $\exists x(H(x) \wedge F(x))$ | $\exists I 8$ |
| 10 | | $\exists x(H(x) \wedge F(x))$ | $\exists E 2, 3-9$ |