Written HW1 – MATH 2502 Spring 2021
Due by 18 January for timely completion credit

These problems will test your abilities of manipulating integral substitutions. The two functions that will be investigated below cannot be expressed in a simpler way — there are (provably!) no anti-derivatives of the functions being integrated to apply the fundamental theorem of calculus to. Thus, the representation using integrals as "an area function" is the best way to express them! Nonetheless, integrals that are encountered in scientific investigations sometimes reduce to these functions.

1. The so-called "sine integral function" Si is a special function that has application to signal processing (a field of electrical engineering which deals with signals of all kinds). It is defined by the formula

$$\text{Si}(x) = \int_0^x \frac{\sin(t)}{t}\,dt.$$

   The problem: Express the integral $\int_0^x \frac{\sin(7t)}{t}\,dt$ in terms of Si.

2. The so-called "error function" erf is a special function that has important application in the theory of statistics (it is essentially the "cdf" of the normal distribution). The error function is defined by the formula

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt.$$

   The problem: Express the definite integral $\int_0^x te^{-t^4}\,dt$ in terms of erf. (*hint: make the substitution* $u = t^2$)