Written HW15 – MATH 2501 Fall 2021

Due by 27 October for timely completion credit

1. Compute $\lim_{x\to-\infty} \frac{x^4+x^3+2000x+1}{-10x^4-300000x+12}$

2. Compute $\lim_{x\to\infty} \frac{x}{\sqrt{x}\ln(x)^2}$

3. Recall the Stirling approximation which says that $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ (*note: as discussed in class, this means you can replace* $n!$ *with* $\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ *inside a limit as* $n\to\infty$).

   Calculate $\lim_{n\to\infty} \frac{n!e^n}{\sqrt{n}n^n}$.

4. It can be shown that a so-called "modified Bessel function of the first kind", $I_1(x)$, has asymptotic formula $I_1(x) \approx \frac{e^x}{\sqrt{2\pi x}}$ (*note: this means you can replace* $I_1(x)$ *with* $\frac{e^x}{\sqrt{2\pi x}}$ *in a limit as* $x\to\infty$).

   Use that fact to compute $\lim_{x\to\infty} e^{-x}I_1(x)$.

   (*another note: "Bessel functions" often appear when solving the "wave equation" on a circular membrane which describes, for instance, the behavior of a snare drum when it is struck by a stick*)