Written HW7 – MATH 3503 Fall 2020

Due by 9 October for timely completion credit

Suppose that the density (measured in appropriate units, e.g. $\frac{\text{grams}}{\text{cm}^2}$) of a flat object D in the plane, at the point (x, y), is given by the function $\rho(x, y)$. The mass of the object is then given by $m = \iint_D \rho(x, y)\mathrm{d}A$. There are two so-called "moments" – a "moment about the x-axis", M_x, given by $M_x = \iint_D y\rho(x, y)\mathrm{d}A$ and a "moment about the y-axis", M_y, given by $M_y = \iint_D x\rho(x, y)\mathrm{d}A$.

The "center of mass" of the object D is the point $(\hat{x}, \hat{y})$ on D, where $\hat{x} = \frac{M_y}{m}$ and $\hat{y} = \frac{M_x}{m}$. The center of mass is the unique location where you can place your finger and have the object balance perfectly (in other words, the object "pretends" that all of its mass is concentrated at this point).

1. A flat object is defined as the region boudned by the curves $y = e^x$, $y = 0$, $x = 0$, and $x = 1$ has density function $\rho(x, y) = y$. Find its center of mass. Include in your answer a plot of this object and put a point at the location of the center of mass (use Desmos, as I did in class).

2. A flat object occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant and has density function $\rho(x, y) = \sqrt{x^2 + y^2}$. Find its center of mass. Include in your answer a plot of this object and put a point at the location of the center of mass (use Desmos, as I did in class).