Written HW12 – MATH 3503 Fall 2020
Due by 20 November for timely completion credit

Suppose S is a surface parametrized by $\vec{r}(u,v)$ for $(u,v) \in D$. The scalar surface integral of a function f is

$$\iint_S f \mathrm{d}S = \iint_D f(\vec{r}(u,v)) \, \|\vec{r}_u \times \vec{r}_v\| \, \mathrm{d}A$$

Suppose further that $\vec{F}$ is a three-dimensional vector field. The vector field surface integral of $\vec{F}$ over S is

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{S} = \iint_D \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v)\mathrm{d}A,$$

where $\vec{r}_u \times \vec{r}_v$ has the appropriate orientation (note: depending on the specified orientation in the problem below, you may instead need to use $\vec{r}_v \times \vec{r}_u$ – check!!)

1. Calculate $\displaystyle\iint_S x^2yz\mathrm{d}S$, where S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $0 \leq x \leq 3$, $0 \leq y \leq 2$.
2. Calculate $\displaystyle\iint_S \vec{F} \cdot \mathrm{d}\vec{S}$ where $\vec{F} = \langle x, y, z^4 \rangle$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ with downward orientation (i.e. normal vectors point away from the surface).