Parametric Surfaces

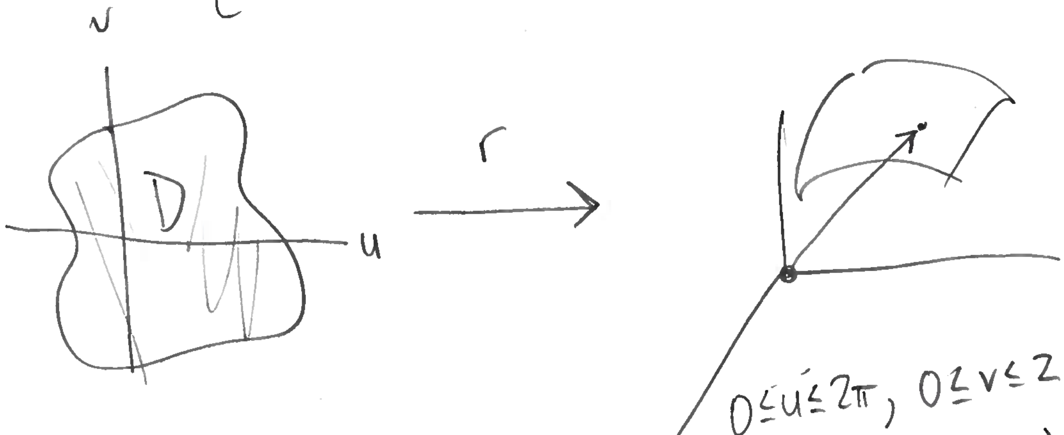
So far we have represented space curves

$$\begin{cases} \vec{r}(t) = \langle x(t), y(t), z(t) \rangle \\ a \le t \le b \end{cases}$$

$$\vec{r} : \mathbb{R} \to \mathbb{R}^n$$

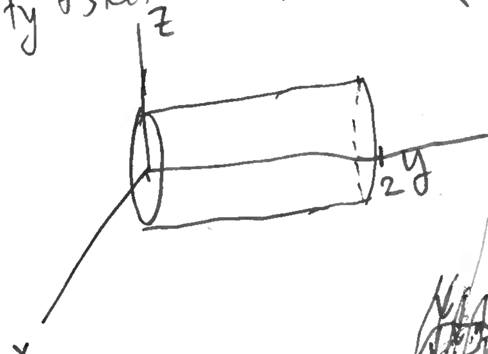
Now: Upgrade domain of the parametrization

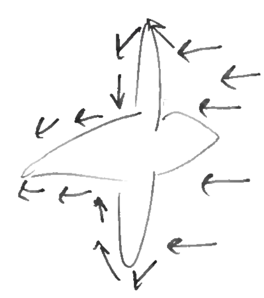
$$\begin{cases} \vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle \\ (u,v) \in D \end{cases}$$



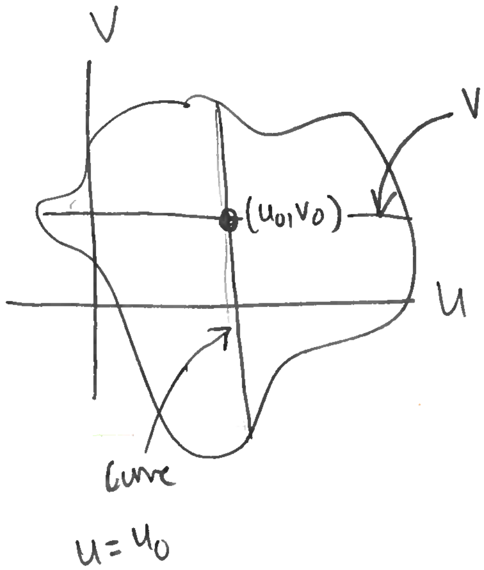
Ex: Identify & sketch $\vec{r}(u,v) = \langle 2\cos(u), v, 2\sin(u) \rangle$, $0 \le u \le 2\pi$, $0 \le v \le 2$

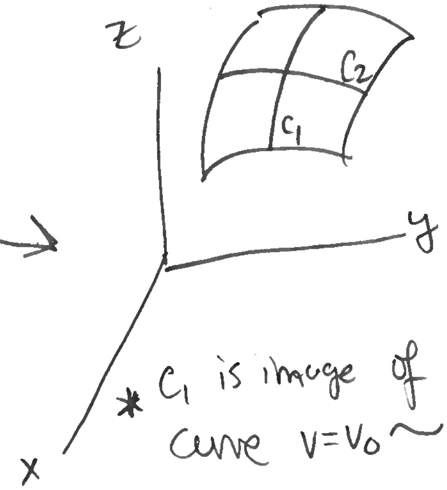
look like circles





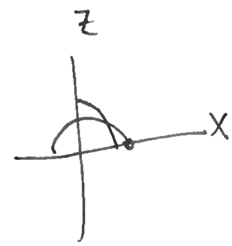
Two useful types of curves



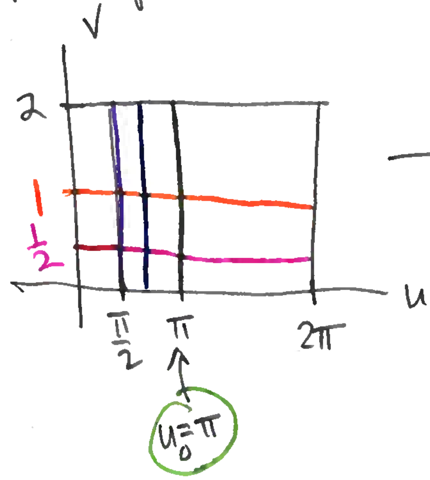


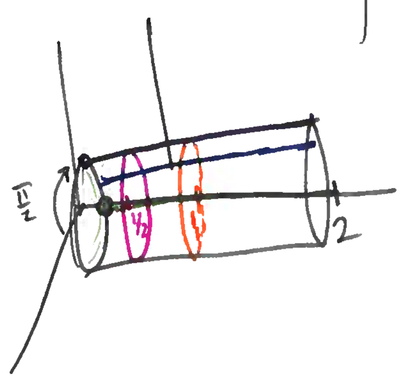
* C_1 is image of curve $v=v_0$ ~ $\vec{r}(u, v_0)$ (free: u)

* C_2 is image of curve $u=u_0$ ~ $\vec{r}(u_0, v)$ (free: v)



Ex: From previous example:





Ex: Plot $\vec{r}(u,v) = \langle (2+\sin(v))\cos(u), (2+\sin(v))\sin(u), u+\cos(v) \rangle$

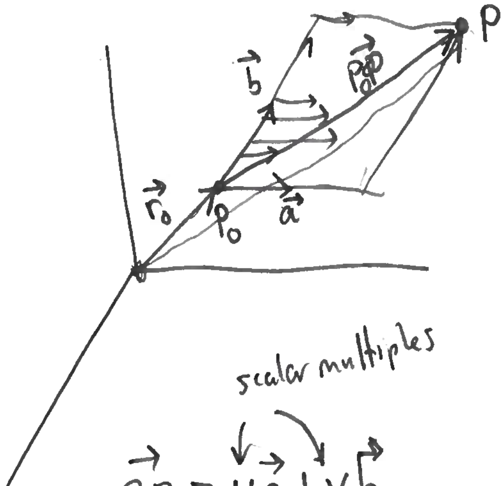
$0 \le u \le 4\pi, \quad 0 \le v \le 2\pi$

Looks cool!

Generally: given eqt $\to$ draw surface ~ easy!

given surface $\to$ find eqt ~ hard!

<u>Ex</u>: Find parametrization of plane passing thru P_0 with offset vector $\vec{r}_0$ that contains two nonparallel vectors $\vec{a}$ and $\vec{b}$.



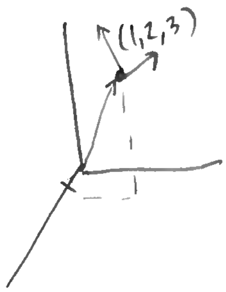
scalar multiples

Vector $\vec{P_0P} = u\vec{a} + v\vec{b}$

If $\vec{r}$ is the vector for P from origin,

$$\vec{r} = \vec{r}_0 + u\vec{a} + v\vec{b}$$

<u>Ex</u>: Parametrize plane containing $(1,2,3)$ containing vectors $\vec{a} = \langle 3,1,-2 \rangle$ and $\vec{b} = \langle 1,1,0 \rangle$



$\Downarrow$

$$\vec{r}(u,v) = \langle 1,2,3 \rangle + u\langle 3,1,-2 \rangle + v\langle 1,1,0 \rangle$$
$$= \langle 1+3u+v,\ 2+u+v,\ 3-2u \rangle$$